Аналитический сигнал, томографическое представление и его применения для анализа медицинских сигналов

Ю.М. Белоусов, Н.А. Елкин,
А.М. Кончаков, В.И. Манько

Tomograms and other transforms: a unified view

M A Man'ko[1,2], V I Man'ko[1,2] and R Vilela Mendes[2,3]

J. Phys. A: Math. Gen. **34** (2001) 8321–8332 PII: S0305-4470(01)24342-X

Signal recognition and adapted filtering by non-commutative tomography

Carlos Aguirre * R. Vilela Mendes †

arXiv:1211.5986v1 [physics.data-an] 26 Nov 2012

Signals on graphs: Transforms and tomograms

R. Vilela Mendes [a,*], Hugo C. Mendes [b], Tanya Araújo [c]

Physica A 450 (2016) 1–17

$$f(t) = \langle t|f\rangle. \qquad \hat{t}f(t) = tf(t),$$

$$[\hat{t}, \widehat{\omega}] = \mathrm{i}, \qquad \widehat{\omega}f(t) = -\mathrm{i}\frac{\partial f(t)}{\partial t}.$$

$$\langle t|\omega\rangle = \frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}\omega t}.$$

Действительному сигналу $f(t)$ ставится в соответствие так называемый "*аналитический сигнал*" $f_a(t) = \langle t|f_a\rangle$ по следующему правилу. Рассмотрим преобразование Фурье действительной функции[2]:

$$\langle\omega|f\rangle = \frac{1}{2\pi}\int \mathrm{d}t f(t)\mathrm{e}^{-\mathrm{i}\omega t} \equiv \int \mathrm{d}t\langle\omega|t\rangle\langle t|f\rangle,$$

Комплексный аналитический сигнал $f_a(t)$ определяется "волновой функцией" в частотном представлении как

$$\langle\omega|f_a\rangle = \langle\omega|f\rangle\theta(\omega).$$

Здесь функция Хевисайда

$$\theta(\omega) = \frac{1}{2}\left(1 + \frac{|\omega|}{\omega}\right) = \begin{cases} 1, & \omega > 0, \\ 0, & \omega < 0. \end{cases}$$

Таким образом, в разложении Фурье аналитического сигнала $f_a(t)$ при $\omega > 0$ его фурье-компоненты совпадают с фурье-компонентами действительного сигнала $f(t)$, а для отрицательных частот $\omega < 0$ равны нулю.

"Волновая функция" аналитического сигнала $|f_a\rangle$ во временном представлении определяется как

$$\langle t|f_a\rangle = f_a(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}\omega\langle\omega|f\rangle\theta(\omega)\mathrm{e}^{\mathrm{i}\omega t} = \frac{1}{\sqrt{2\pi}}\int\limits_0^\infty \mathrm{d}\omega\langle\omega|f\rangle\mathrm{e}^{\mathrm{i}\omega t},$$

$$f_a(t) = \int_{-\infty}^{\infty} \mathrm{d}t' \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} f(t') \mathrm{e}^{\mathrm{i}\omega(t-t')}.$$

$$\delta_-(t) = \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} f(t') \mathrm{e}^{\mathrm{i}\omega t} = \frac{1}{2}\delta(t) + \frac{\mathrm{i}}{\pi}\wp\frac{1}{t},$$

$$f_a(t) = \frac{1}{2} f(t) - \frac{\mathrm{i}}{\pi}\wp \int_{-\infty}^{\infty} \mathrm{d}t' \frac{f(t')}{t-t'}.$$

$$f(t) = f_a(t) + f_a^*(t),$$

$$f_a(t) = \int\limits_{-\infty}^{\infty} \mathrm{d}t' \int\limits_{0}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \big(f_a(t') + f_a^*(t')\big) \mathrm{e}^{\mathrm{i}\omega(t-t')} =$$

$$= \int\limits_{-\infty}^{\infty} \mathrm{d}t' \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} f_a(t') \mathrm{e}^{\mathrm{i}\omega(t-t')}.$$

$$\frac{1}{2\pi} \int\limits_{0}^{\infty} \mathrm{d}\omega \left[\int\limits_{-\infty}^{\infty} \mathrm{d}t' f_a(t') \mathrm{e}^{\mathrm{i}\omega(t-t')} \right] = f_a(t),$$

поэтому интеграл от аналитического сигнала с обобщенной функцией $\delta_-(t-t')$ и с “полной” δ-функцией совпадают.

Матрица плотности, функция Вигнера и томограмма сигнала

$$\langle f'|f\rangle = \delta(f - f') \quad \text{и} \quad \int \mathrm{d}f |f\rangle\langle f| = \hat{1}.$$

При этом считается, что амплитуда сигнала принимает непрерывные значения в некотором интервале $[f_{\min}, f_{\max}]$, т.е. имеет непрерывный спектр. Если сигнал представлен в цифровом виде, он принимает дискретные значения,

$$\langle f'|f\rangle = \delta_{f,f'} \quad \text{и} \quad \sum_{f=f_{\min}}^{f_{\max}} |f\rangle\langle f| = \hat{1}.$$

$$\rho^s = |f\rangle\langle f|,$$

или во "временном представлении"

$$\rho^s(t,t') = \langle t|f\rangle\langle f|t'\rangle \equiv f(t)f^*(t').$$

Определим теперь функцию Вигнера сигнала как

$$W^s(k,t) = \int\limits_{-\infty}^{+\infty} \mathrm{d}\, x \rho^s \left(t + \frac{x}{2}, t - \frac{x}{2}\right) \mathrm{e}^{-\mathrm{i}kx},$$

$$\rho^s(t,t') = \int\limits_{-\infty}^{+\infty} \frac{\mathrm{d}\, k}{2\pi} W^s \left(k, \frac{t+t'}{2}\right) \mathrm{e}^{\mathrm{i}k(t-t')}.$$

$$f(t)f^*(0) = \int\limits_{-\infty}^{+\infty} \frac{\mathrm{d}\, k}{2\pi} W^s \left(k, \frac{t}{2}\right) \mathrm{e}^{\mathrm{i}kt}.$$

$$f^*(0) = |f(0)|\mathrm{e}^{-\mathrm{i}\varphi(0)},$$

$$f(t)=\mathrm{e}^{\mathrm{i}\varphi(0)}\frac{\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\mathrm{d}\,kW^{s}\left(k,\frac{t}{2}\right)\mathrm{e}^{\mathrm{i}kt}}{\sqrt{\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\mathrm{d}\,kW^{s}(k,0)}}.$$

Теперь можно определить томограмму сигнала

$$w^{s}(x,\mu,\nu)=\iiint\limits_{infty}^{\infty}\frac{\mathrm{d}t\mathrm{d}k\mathrm{d}u}{(2\pi)^{2}}W^{s}(k,t)\mathrm{e}^{-\mathrm{i}u(x-\mu t-\nu k)}$$

$$w^{s}(x,\mu,\nu)=\iiiint\frac{\mathrm{d}t\mathrm{d}k\mathrm{d}u\mathrm{d}y}{(2\pi)^{2}}f(t+y/2)f^{*}(t-y/2)\mathrm{e}^{-\mathrm{i}ky}\mathrm{e}^{-\mathrm{i}u(x-\mu t-\nu k)}.$$

$$\int_{-\infty}^{\infty} \mathrm{d}k \mathrm{e}^{-\mathrm{i}k(y-\nu u)} = 2\pi\delta(y-\nu u),$$

$$w^s(x,\mu,\nu) = \frac{1}{2\pi}\iiint \mathrm{d}t\mathrm{d}u\mathrm{d}y f(t+y/2)f^*(t-y/2)\mathrm{e}^{-\mathrm{i}u(x-\mu t)}\delta(y-\nu u).$$

Выполняя интегрирование по переменной u получаем:

$$w^s(x,\mu,\nu) = \frac{1}{2\pi|\nu|}\iint \mathrm{d}t\mathrm{d}y f(t+y/2)f^*(t-y/2)\mathrm{e}^{-\mathrm{i}\frac{y}{\nu}(x-\mu t)}.$$

$$t = (z+q)/2, \quad y = z-q.$$

Якобиан преобразования (28) равен -1, но поскольку пределы интегрирования распространяются от $-\infty$ до $+\infty$ имеем:

$$w^s(x,\mu,\nu) = \frac{1}{2\pi|\nu|} \iint \mathrm{d}z\mathrm{d}q f(z) f^*(q) \mathrm{e}^{-\mathrm{i}\frac{1}{\nu}\left((z-q)x - \mu(z^2-q^2)/2\right)} =$$

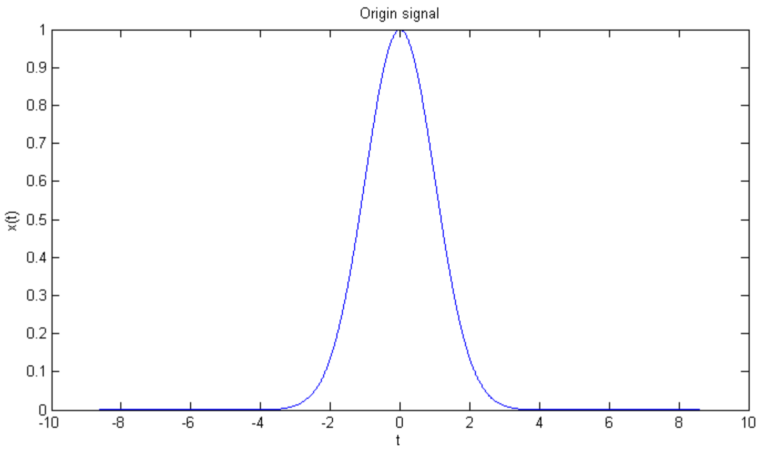
$$= \frac{1}{2\pi|\nu|} \left| \int\limits_{-\infty}^{\infty} \mathrm{d}z f(z) \mathrm{e}^{\frac{\mathrm{i}\mu}{2\nu}z^2 - \frac{\mathrm{i}xz}{\nu}} \right|^2 .$$

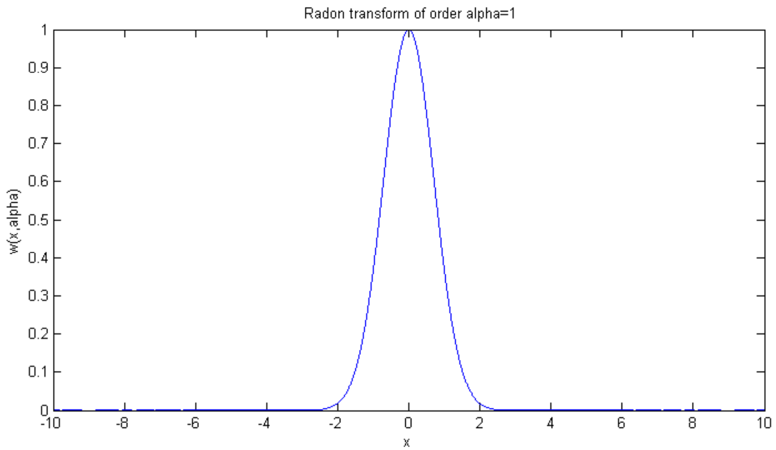
Вводя “привычное” обозначение переменной интегрирования буквой t запишем определение томограммы как

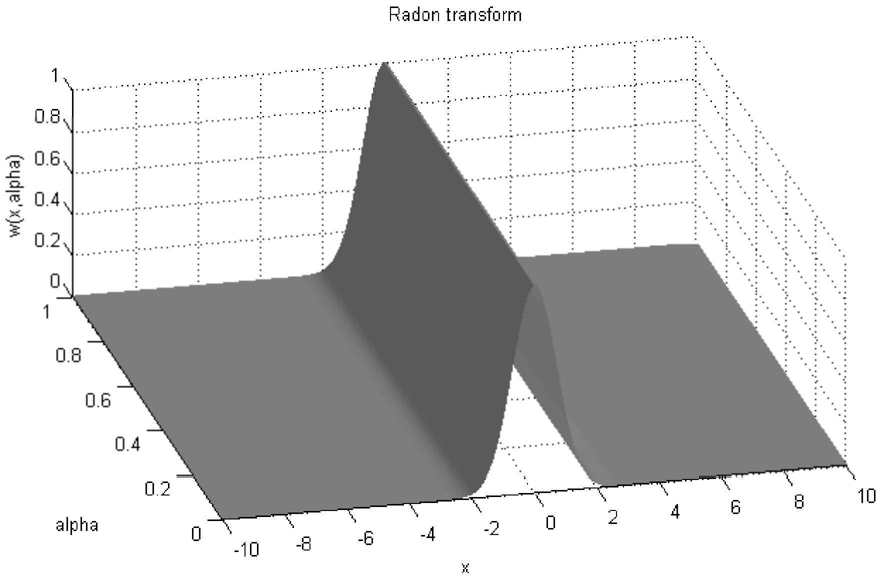
$$w^s(x,\mu,\nu) = \frac{1}{2\pi|\nu|} \left| \int\limits_{-\infty}^{\infty} f(t) \exp\left[\frac{\mathrm{i}\mu}{2\nu}t^2 - \frac{\mathrm{i}xt}{\nu}\right] \mathrm{d}t \right|^2 .$$

Тестирование программы численного счета

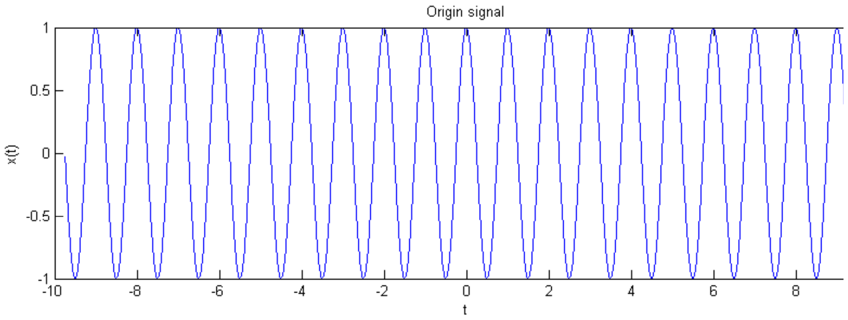
Распределение Гаусса

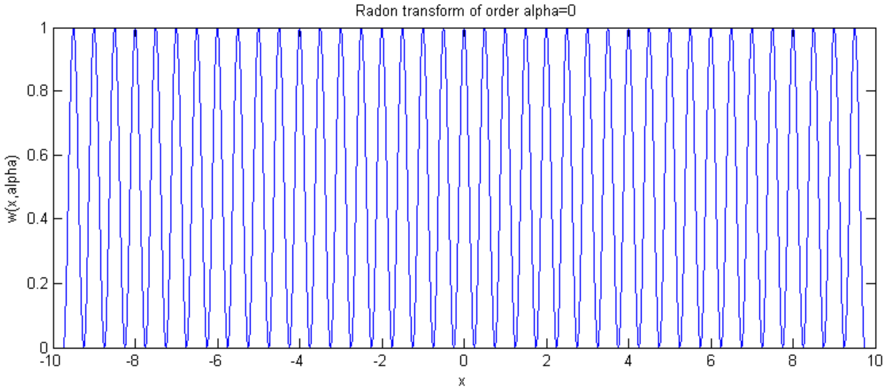


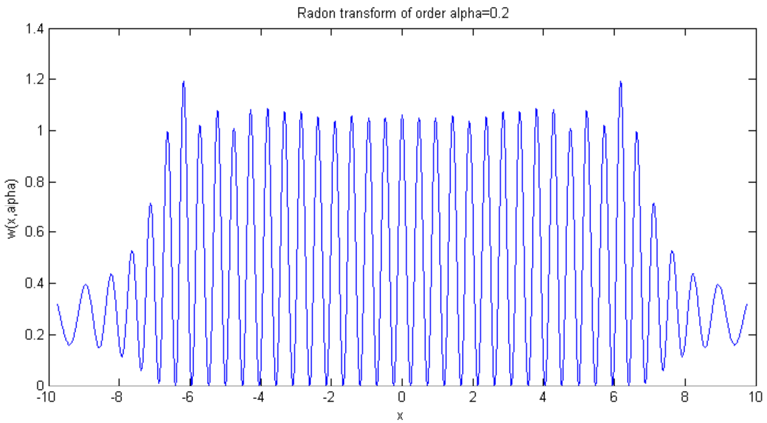


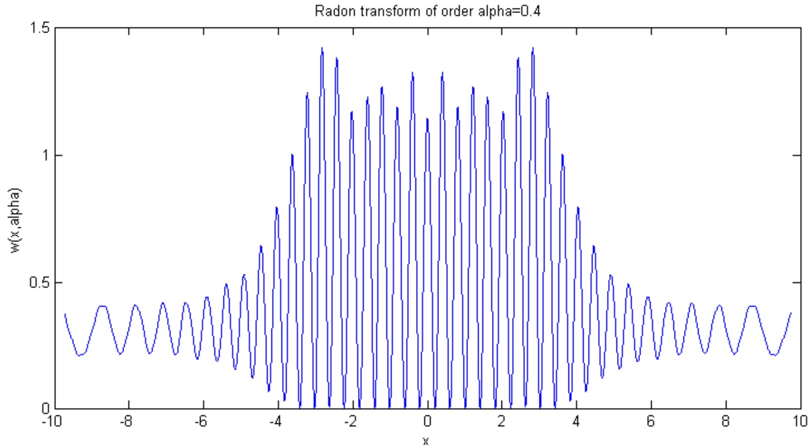


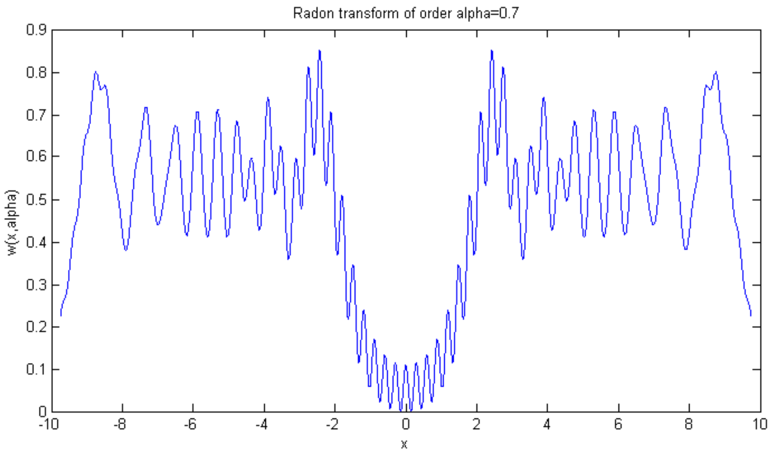
Периодический сигнал

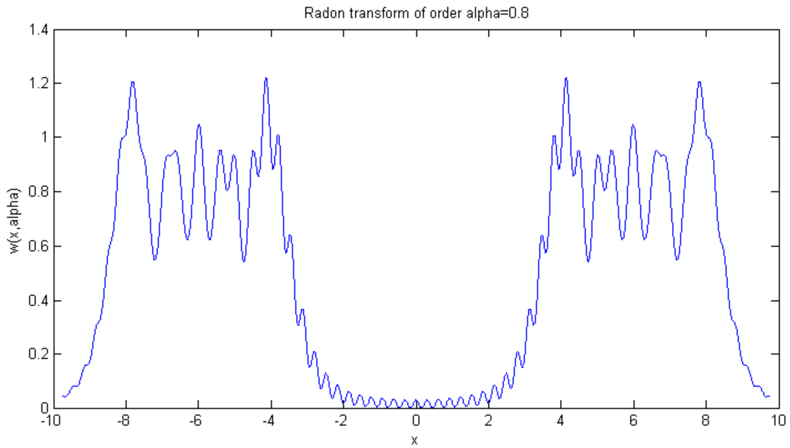


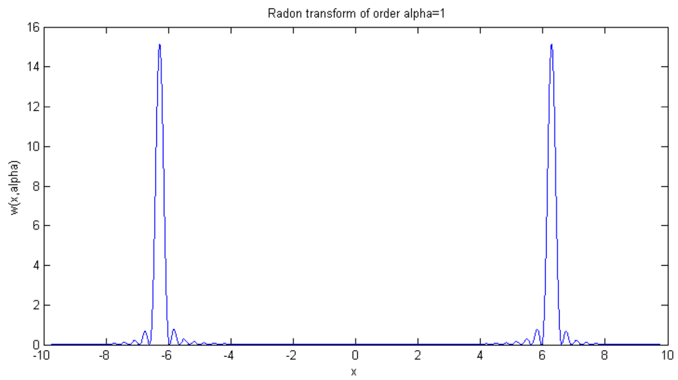








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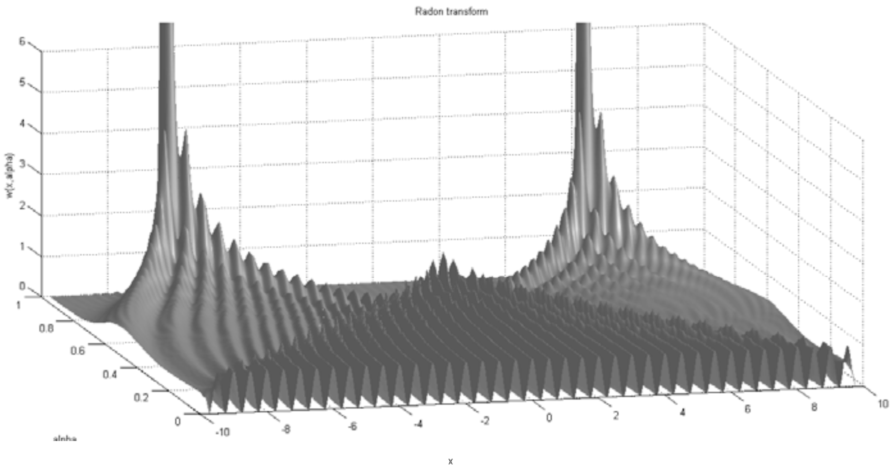
x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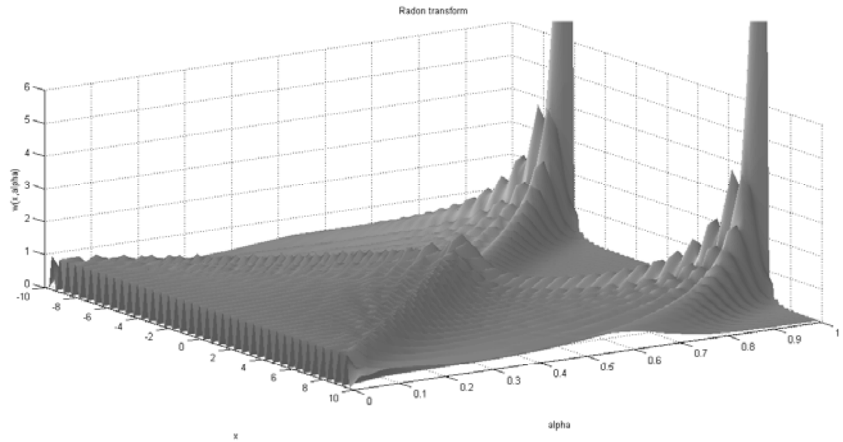
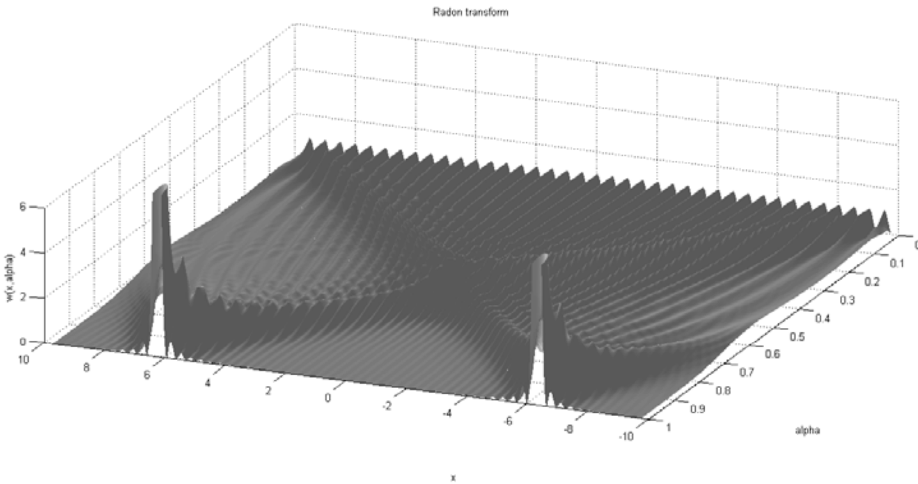
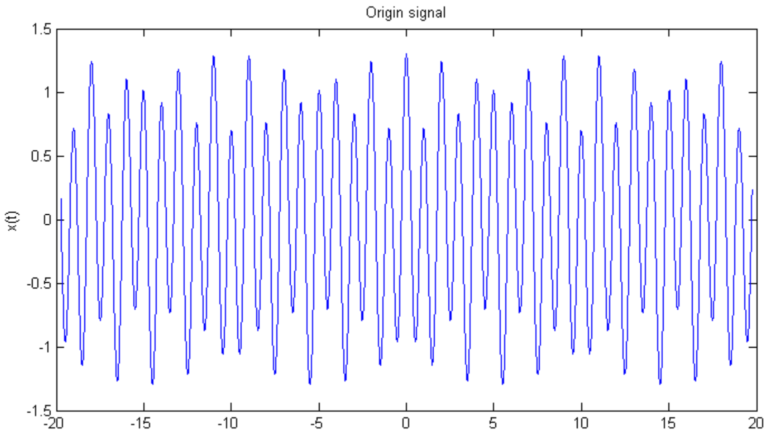
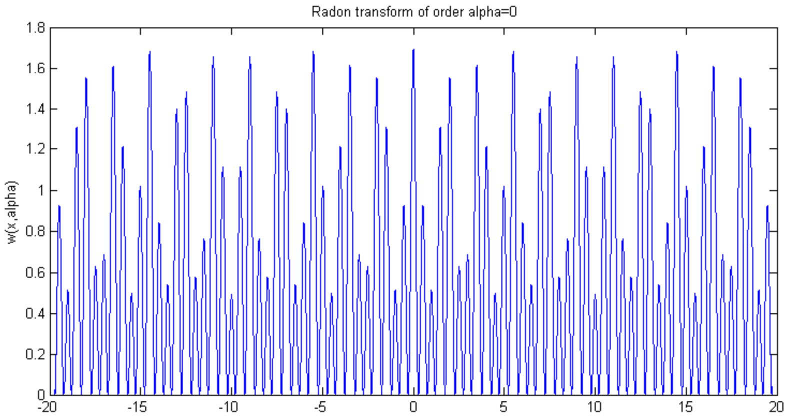
x

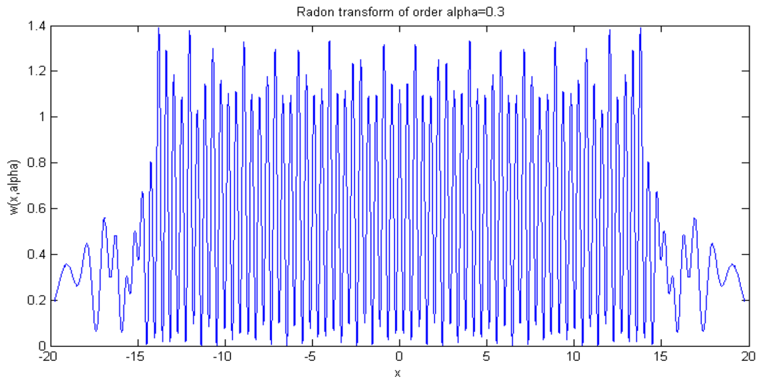

Radon transform
w(x,alpha)
x
alpha

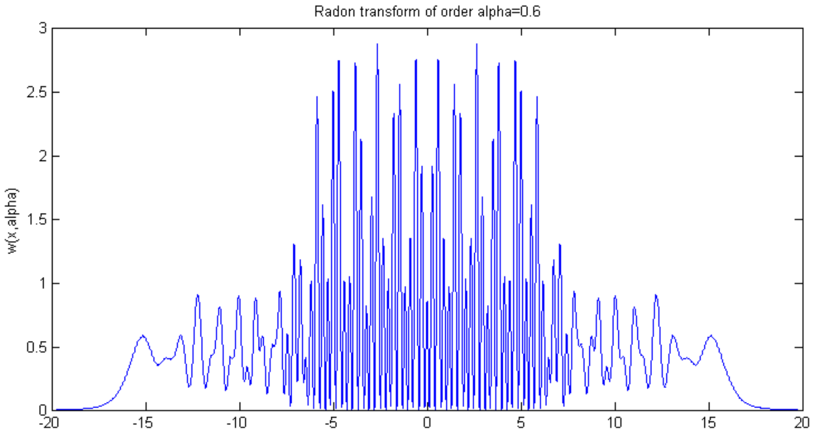

Radon transform
w(x,alpha)
x
alpha

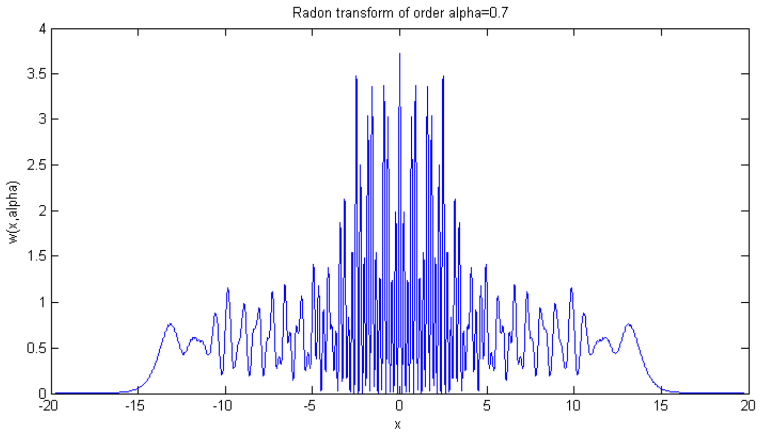
Модулированный периодический сигнал

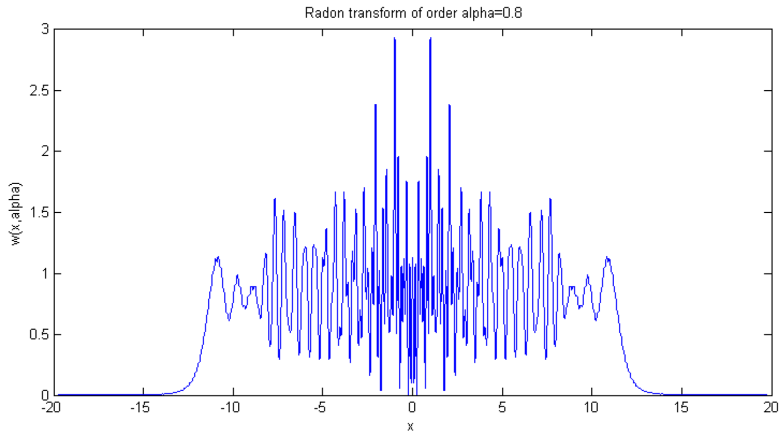


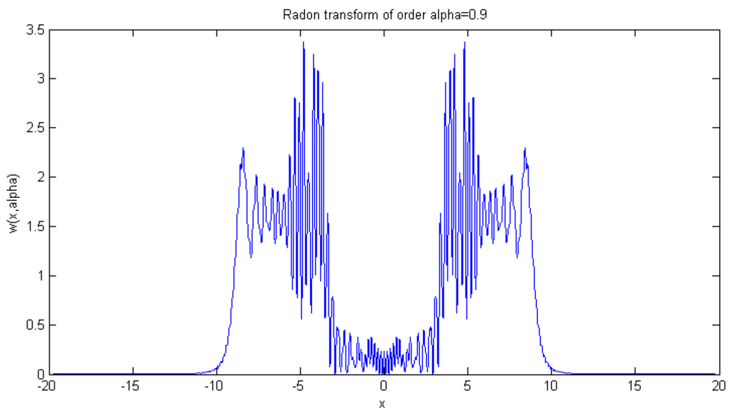








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x
0
0.5
1
1.5
2
2.5
3
3.5
-20
-15
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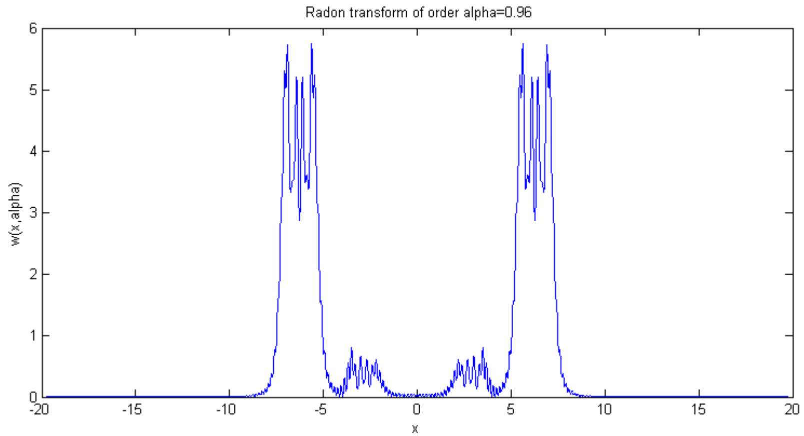
-5
0
5
10
15
20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2
3
4
5
6
-20
-15
-10
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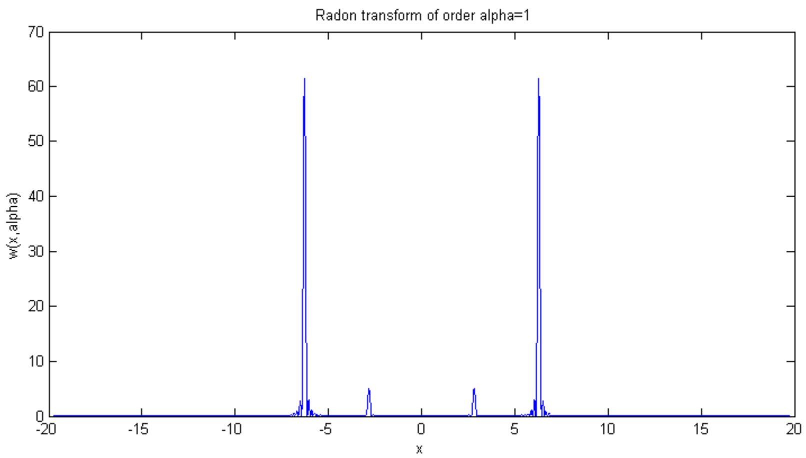
0
5
10
15
20


Radon transform of order alpha=1
w(x,alpha)
x
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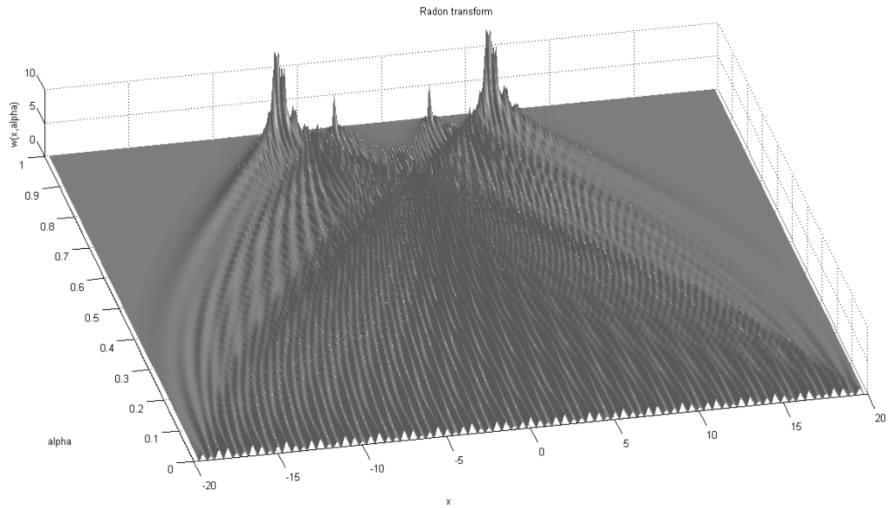
10
20
30
40
50
60
70
-20
-15
-10
-5
0
5
10
15
20


Radon transform
w(x,alpha)
alpha
x
10
5
0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-20
-15
-10
-5
0
5
10
15
20

Анализ медицинских сигналов. ЭКГ

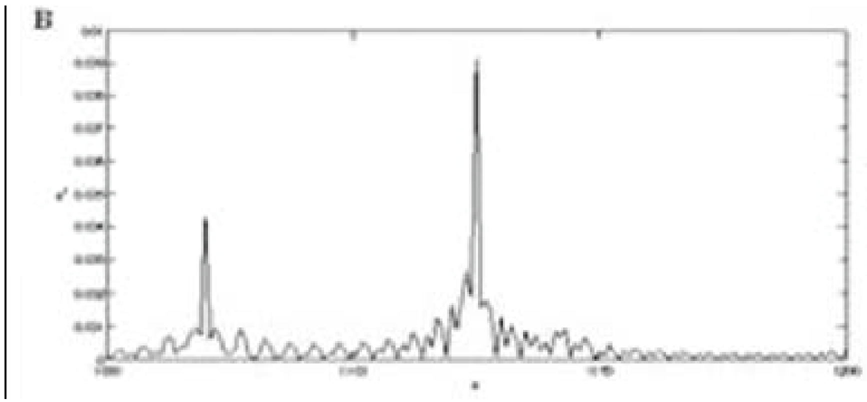
Aguirre *et al. BMC Neuroscience* 2011, **12**(Suppl 1):P297
http://www.biomedcentral.com/1471-2202/12/S1/P297

BMC Neuroscience

POSTER PRESENTATION **Open Access**

Single neuron transient activity detection by means of tomography

Carlos Aguirre*, Pedro Pascual, Doris Campos, Eduardo Serrano



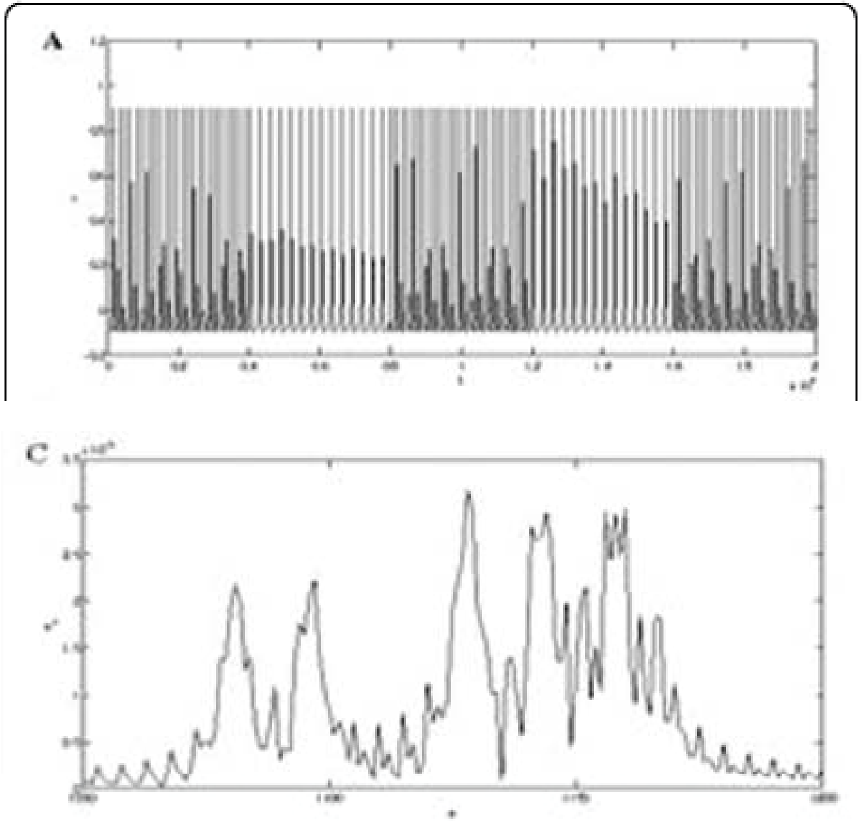
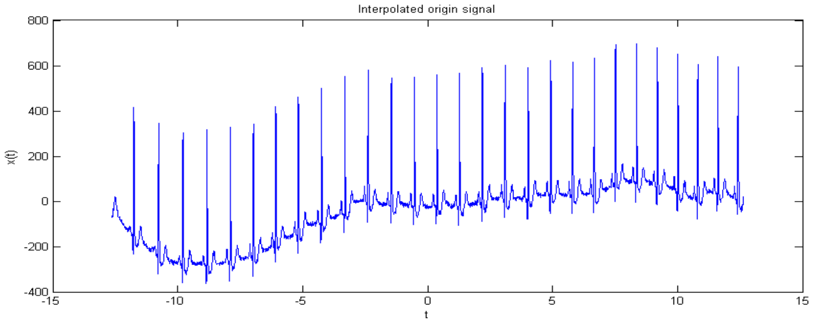
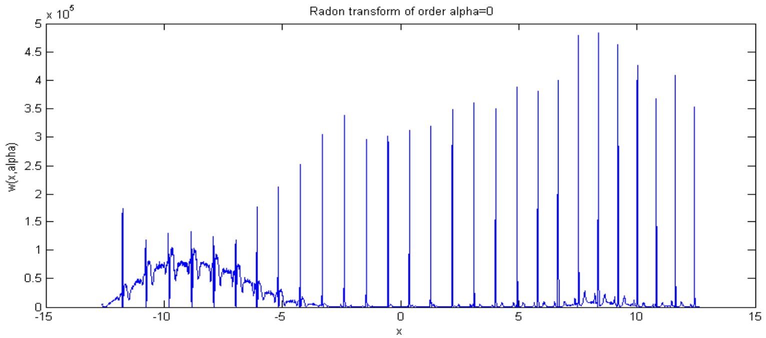


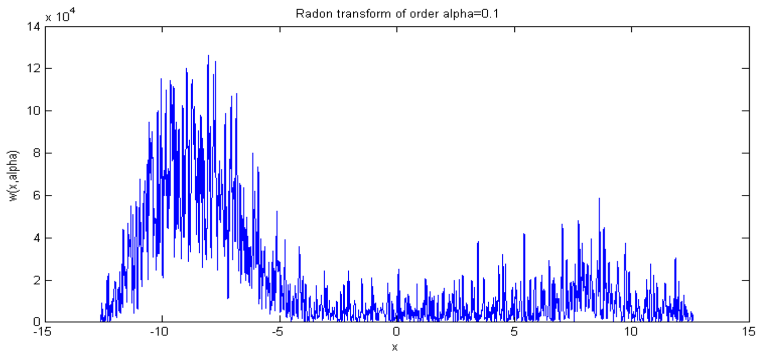
Figure 1 **A.** Neuronal Signal with transient behaviour. **B.** Fourier Transform. **C.** Tomographic Transform.

Кардиограммы

Интервал 250-274







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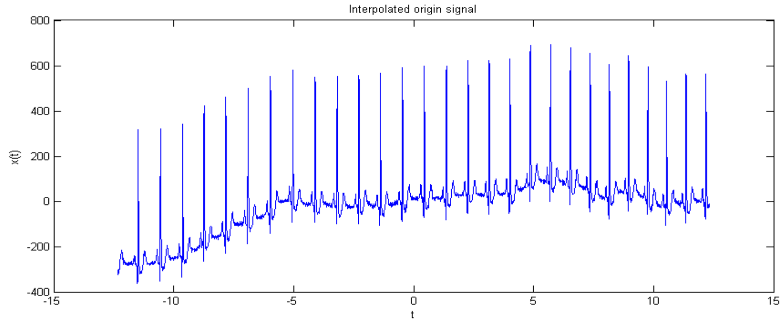

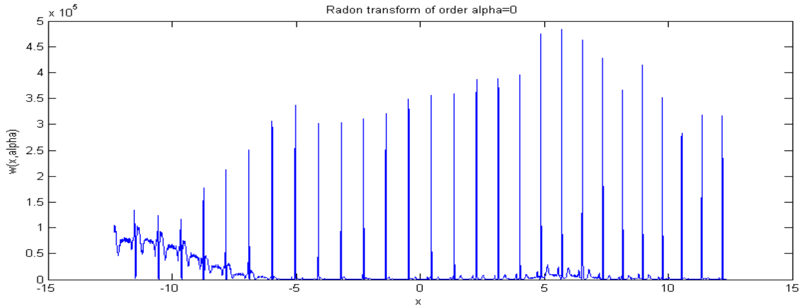

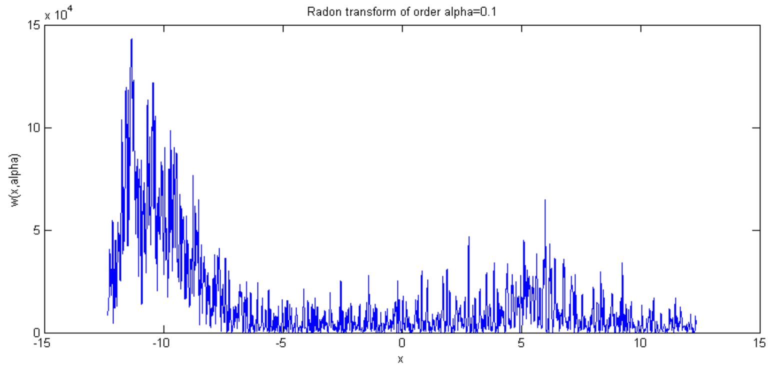

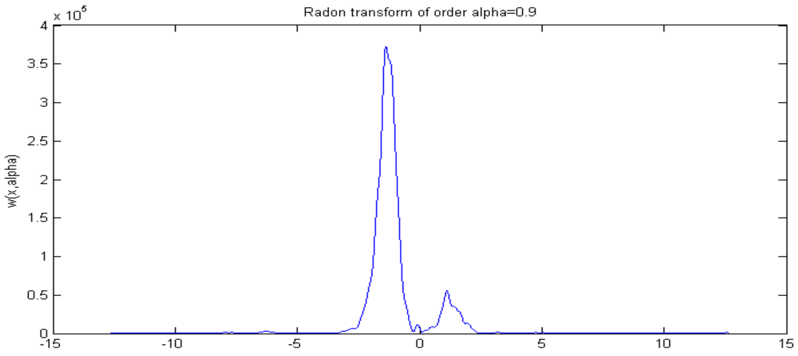
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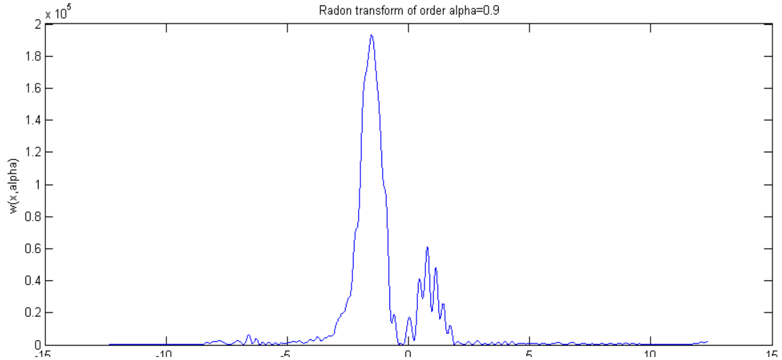
x 10^5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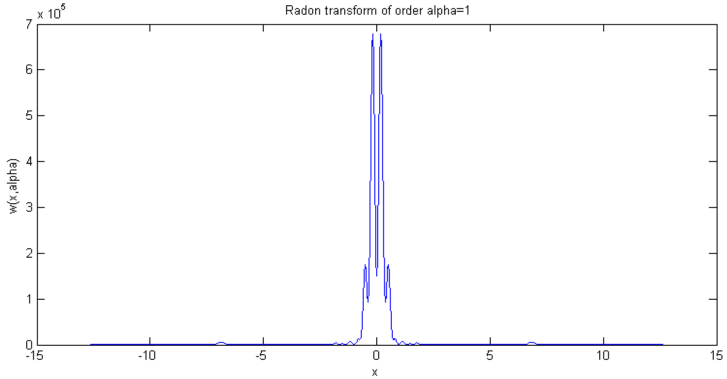
x 10^5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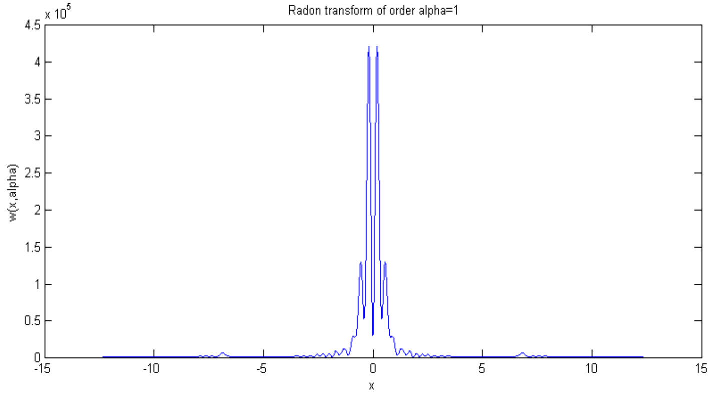
w(x,alpha)
x 10^5
x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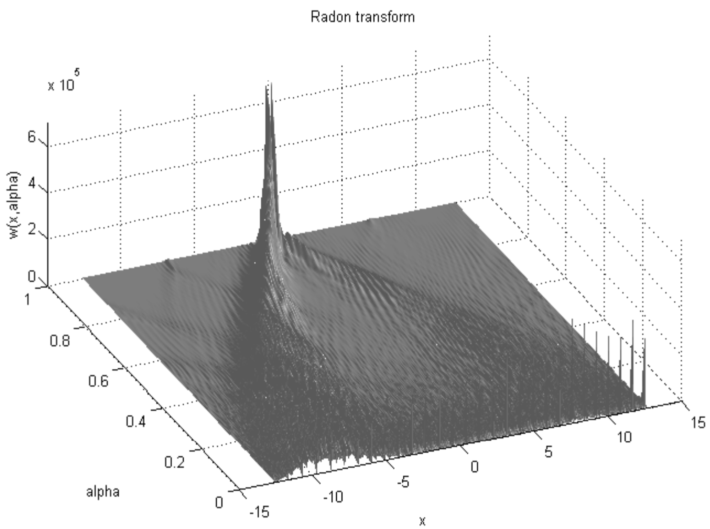
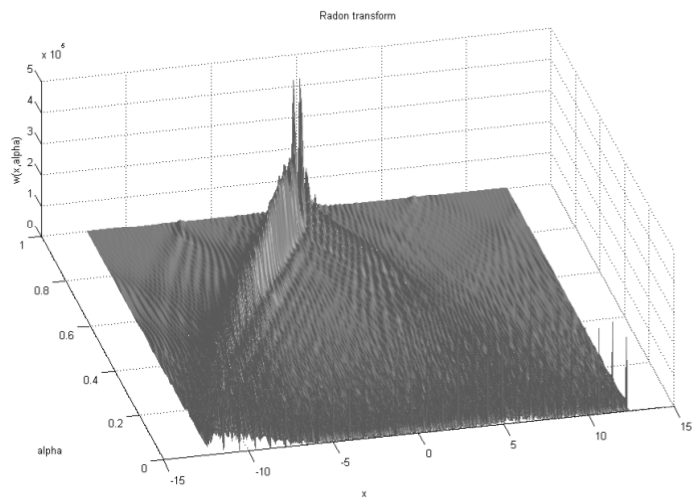
w(x,alpha)
x 10^5
x


Radon transform
w(x,alpha)
x 10^5
alpha
x


Radon transform
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x 10^5
alpha
x

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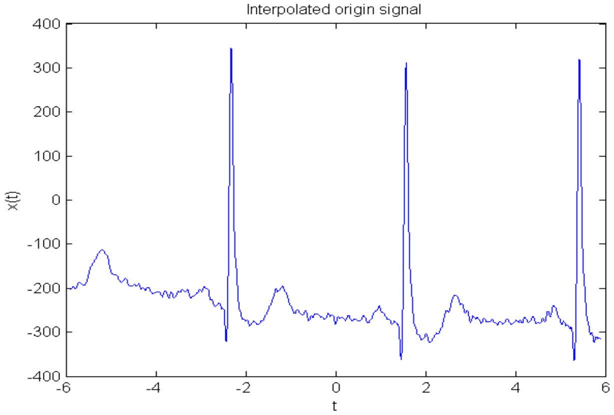

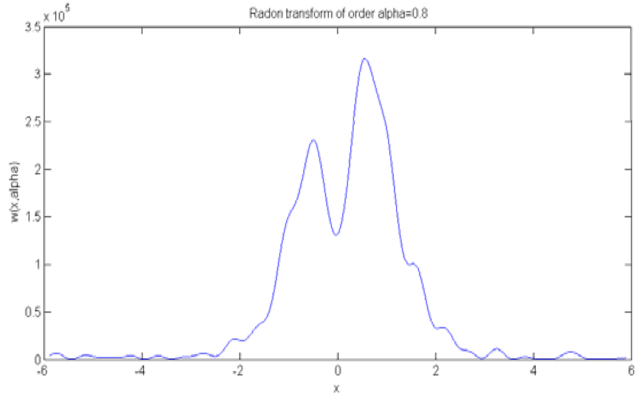

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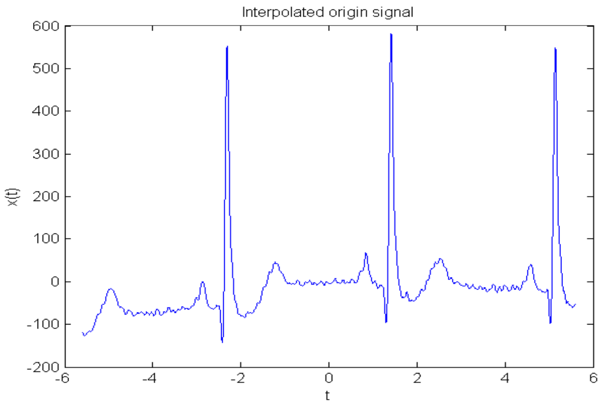
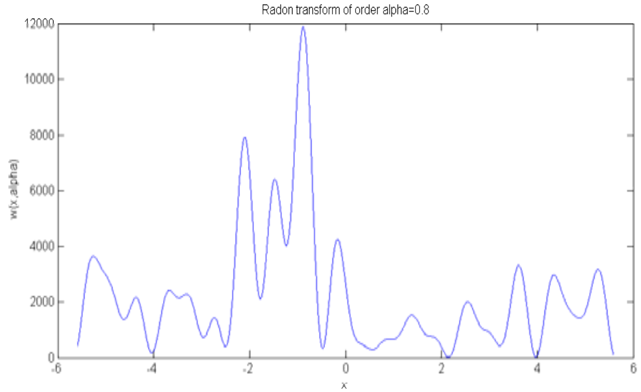
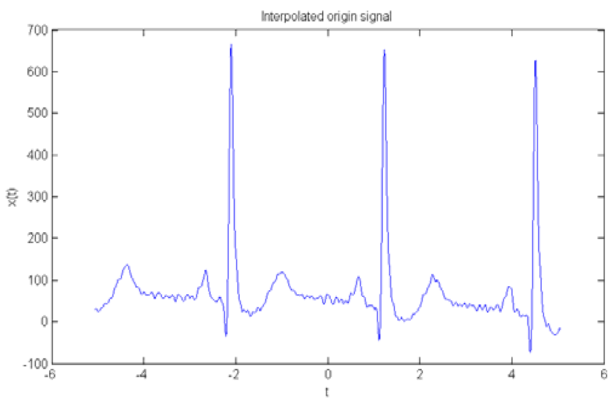
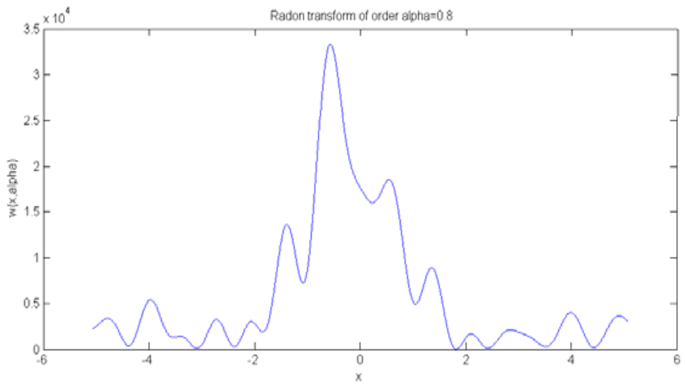
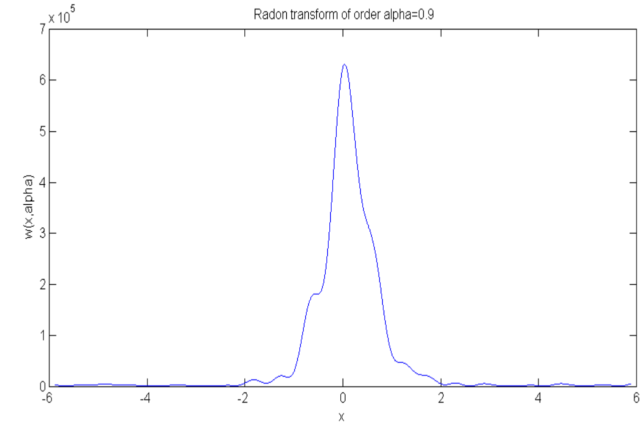


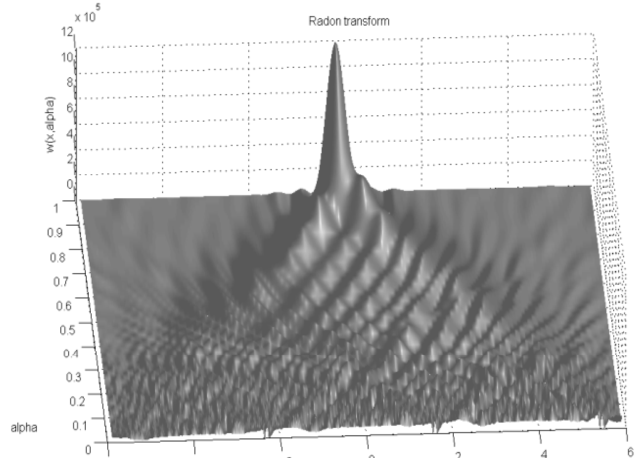


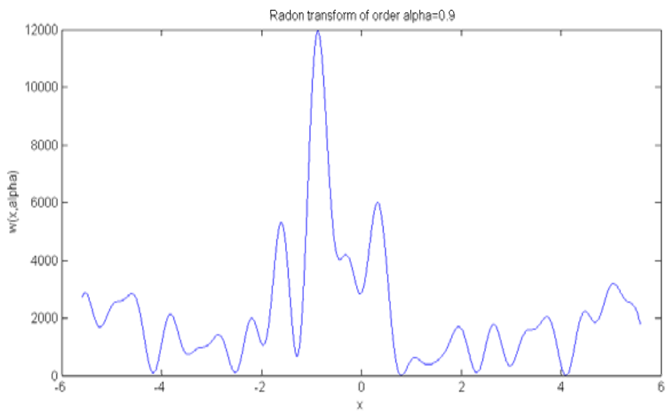
Интервал 270-273

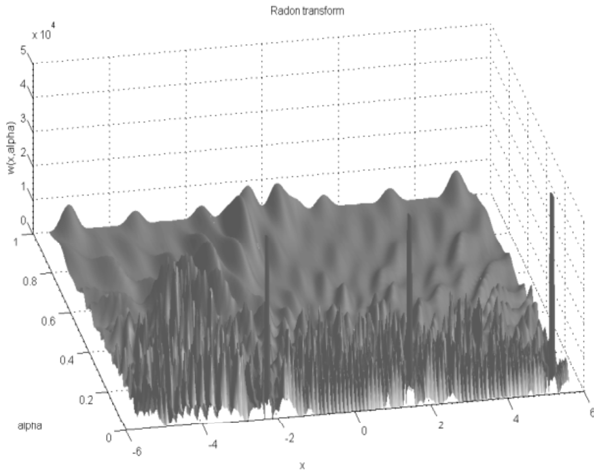


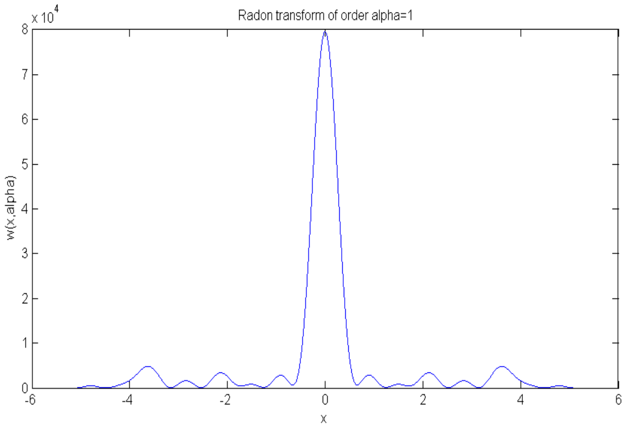


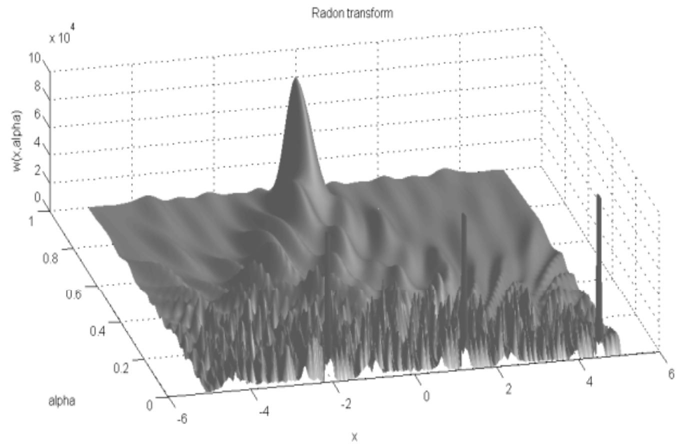

Radon transform of order alpha=0.9
x 10^5
w(x,alpha)
x


Radon transform
x 10^5
w(x,alpha)
alpha

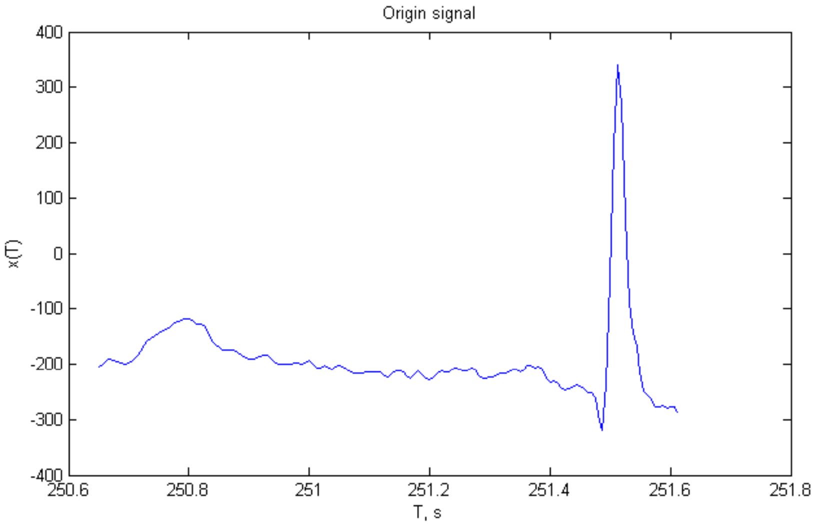

Radon transform of order alpha=0.9
w(x,alpha)
x

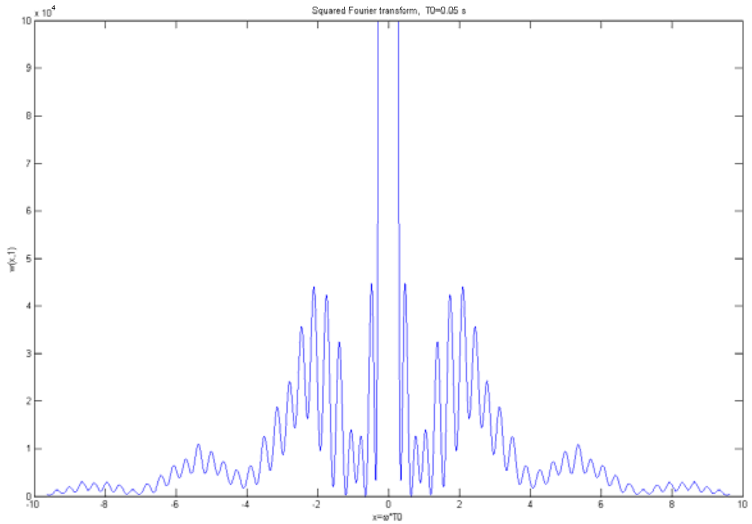

Radon transform
x 10^4
w(x,alpha)
alpha
x

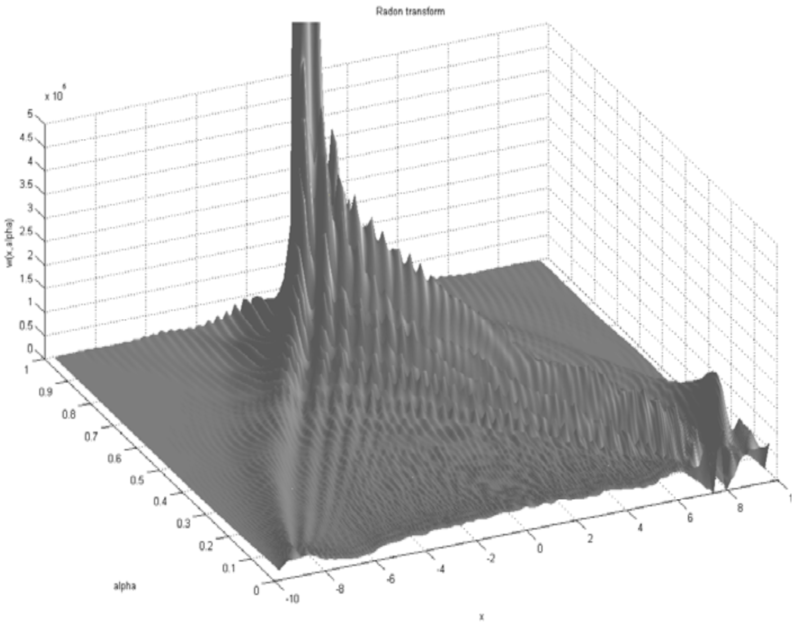

Radon transform of order alpha=1
x 10^4
w(x,alpha)
x

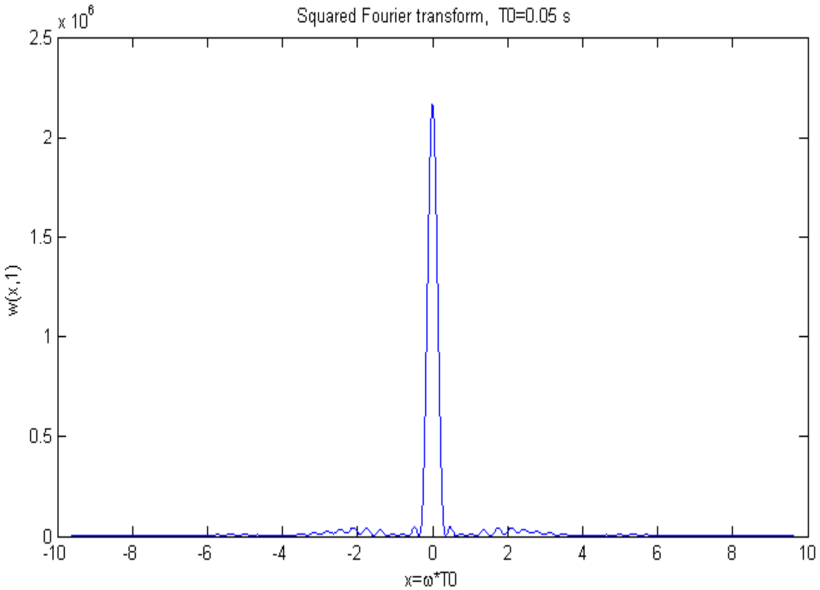

Radon transform
x 10^4
w(x,alpha)
alpha
x

Разбиение интервала 250-253 на три одиночных пульса

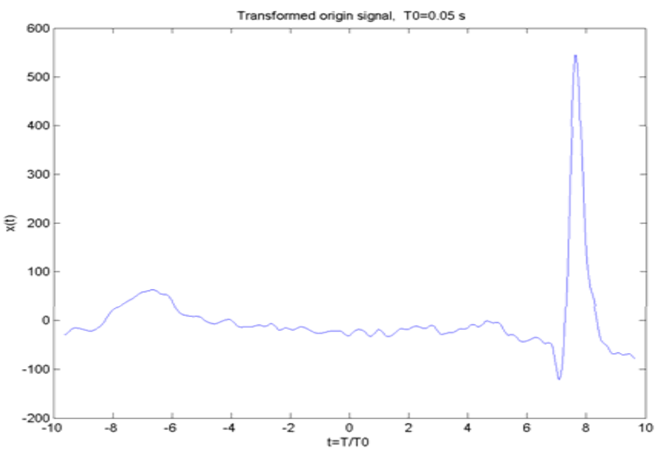


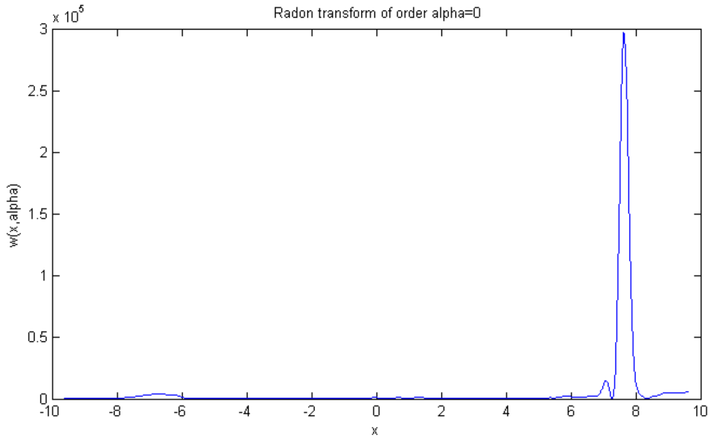




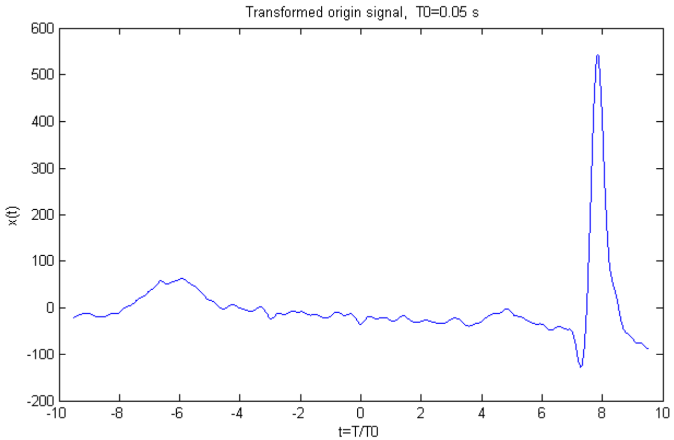


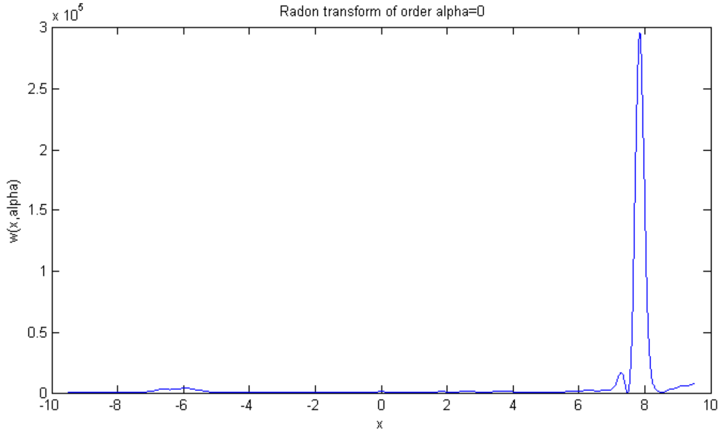
250-251



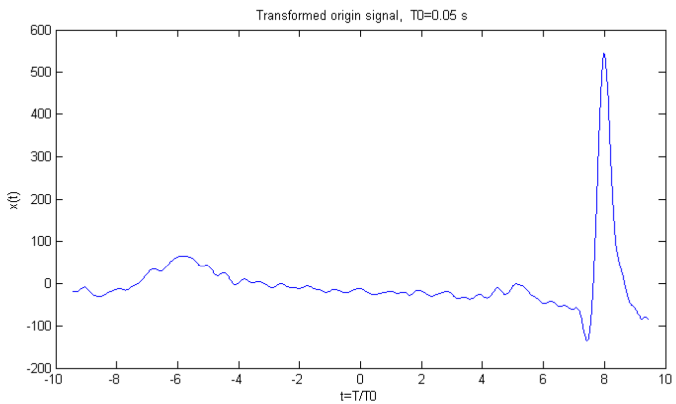


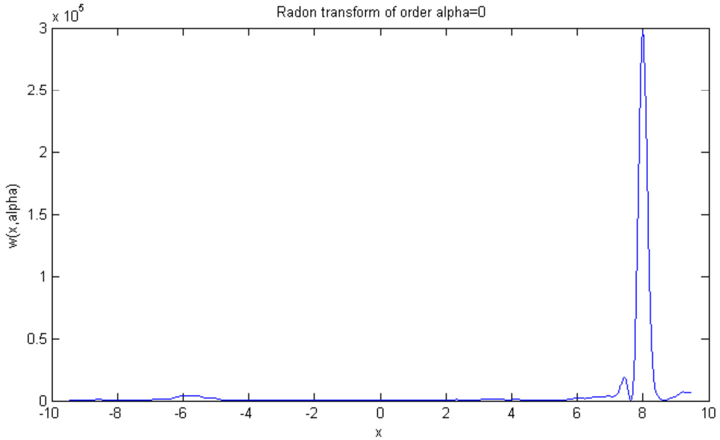
251-252

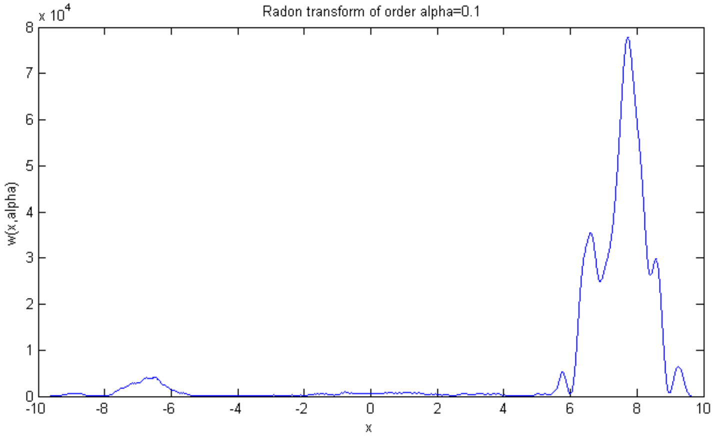


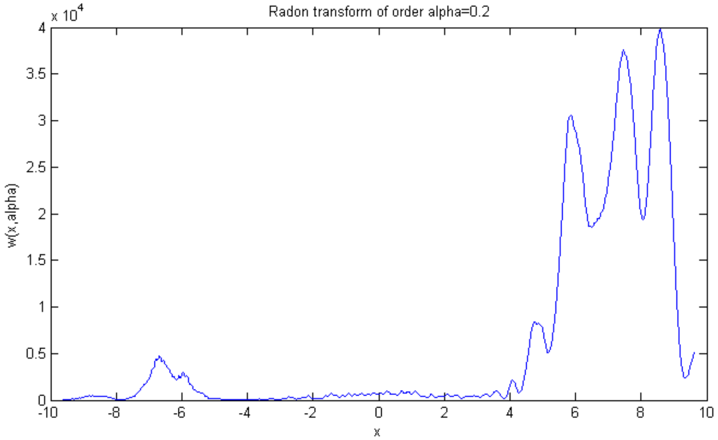


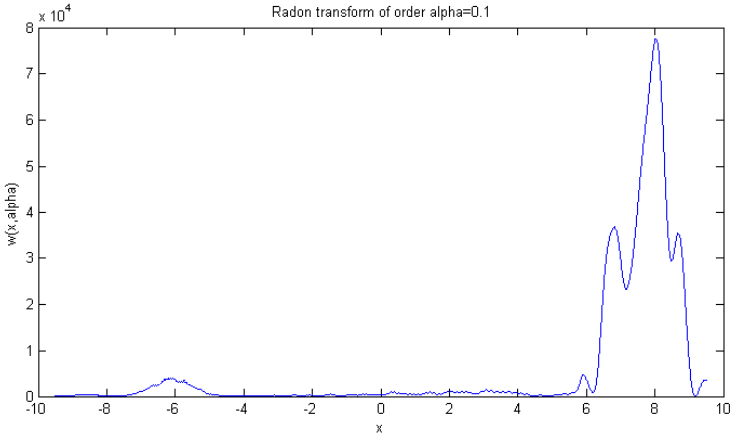
252-253

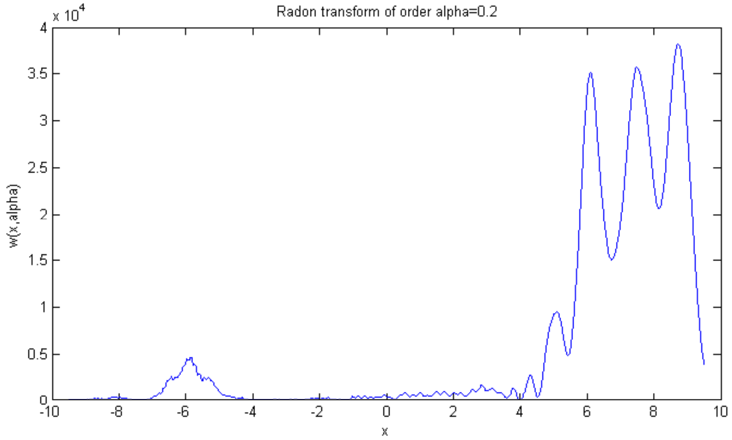


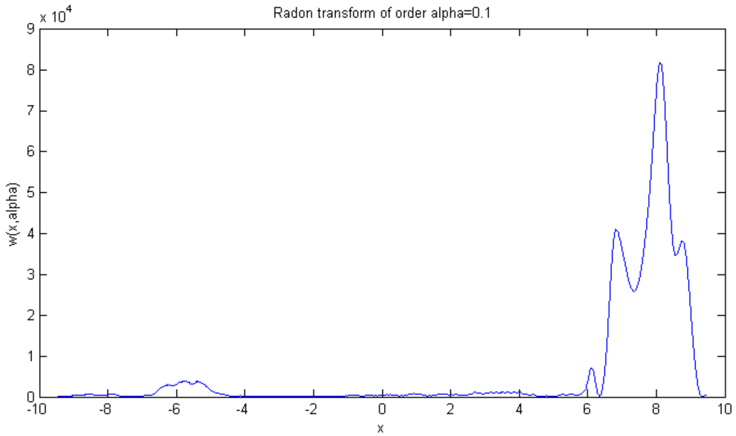


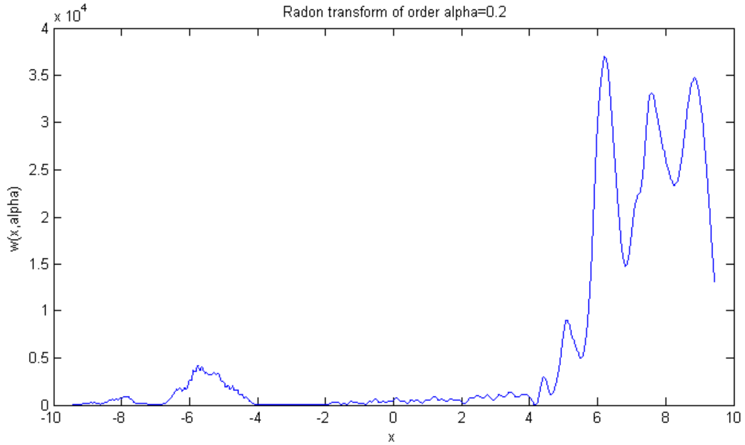

Radon transform of order alpha=0.1
x 10^4
w(x,alpha)
x

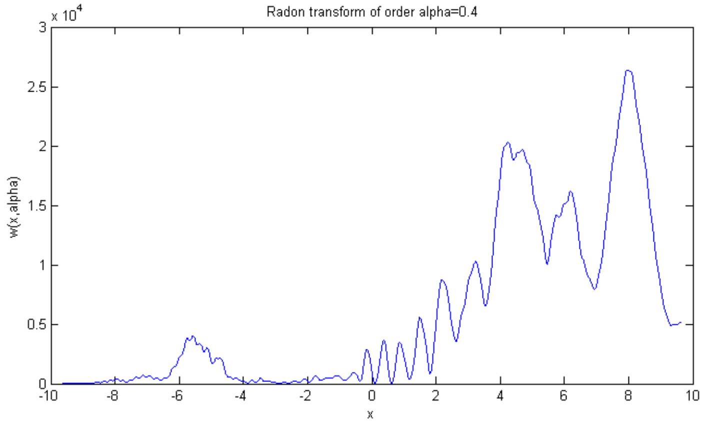

Radon transform of order alpha=0.2
x 10^4
w(x,alpha)
x

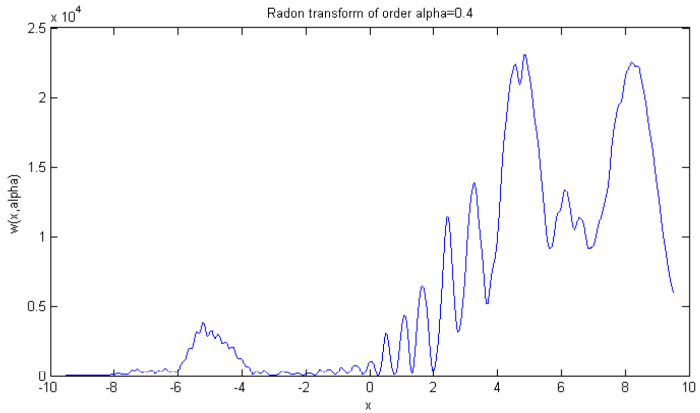

Radon transform of order alpha=0.1
x 10^4
w(x,alpha)
x

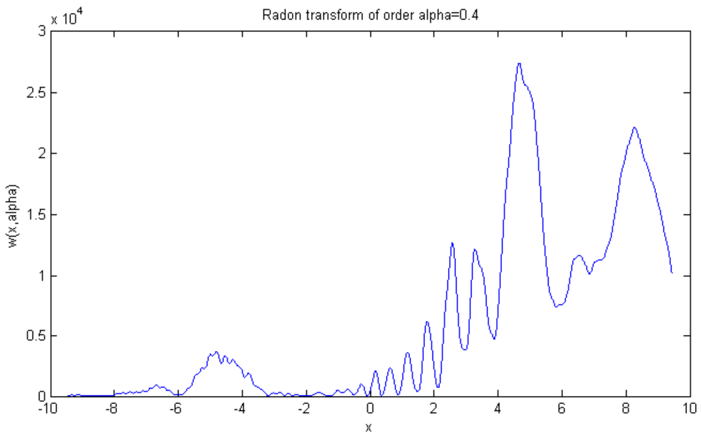

Radon transform of order alpha=0.2
x 10^4
w(x,alpha)
x

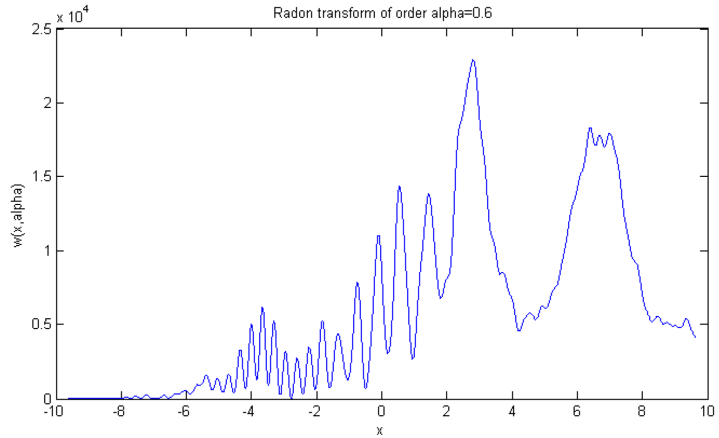

Radon transform of order alpha=0.1
x 10^4
w(x,alpha)
x

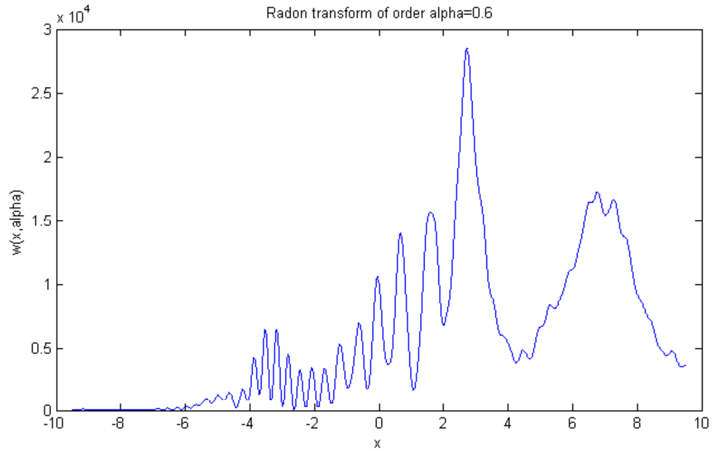

Radon transform of order alpha=0.2
x 10^4
w(x,alpha)
x

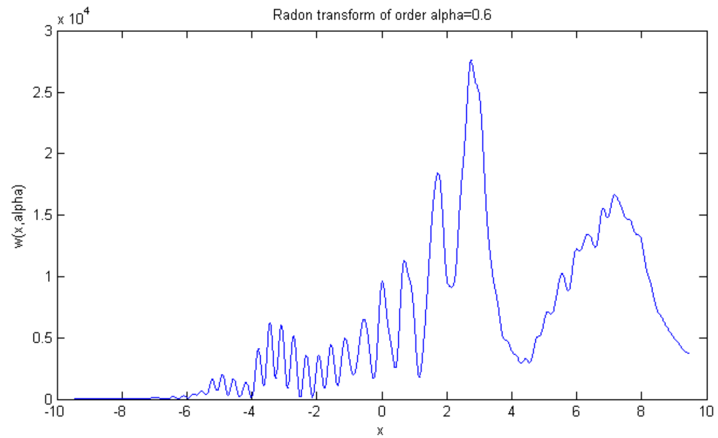

Radon transform of order alpha=0.4
w(x,alpha)
x

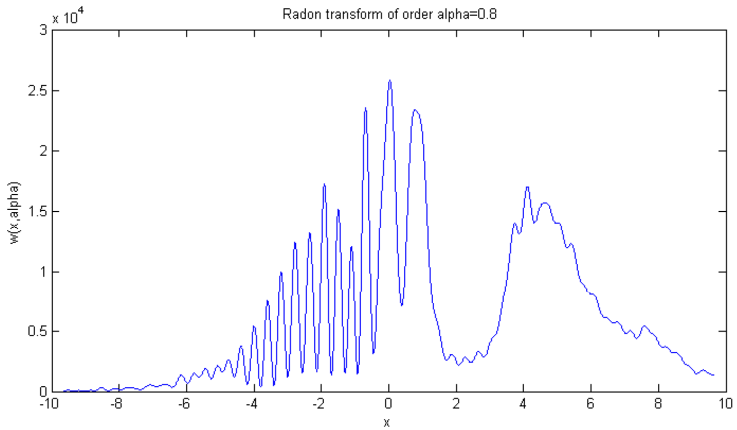

Radon transform of order alpha=0.4
w(x,alpha)
x

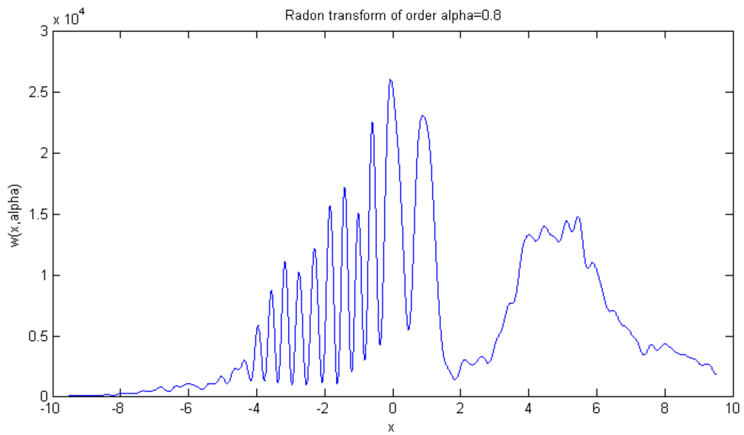

Radon transform of order alpha=0.4
w(x,alpha)
x

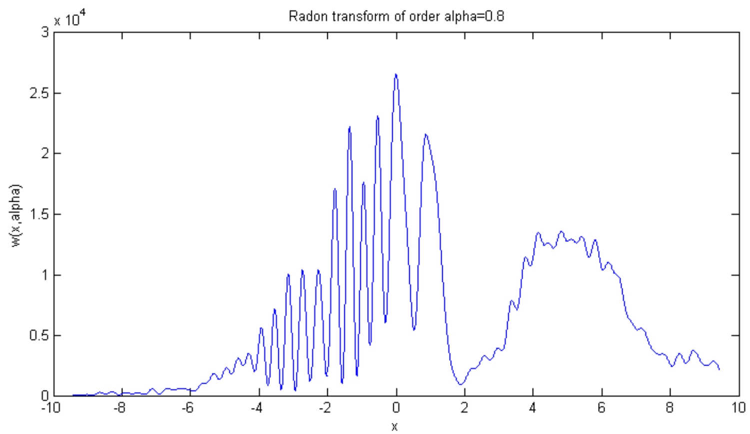

Radon transform of order alpha=0.6
w(x,alpha)
x

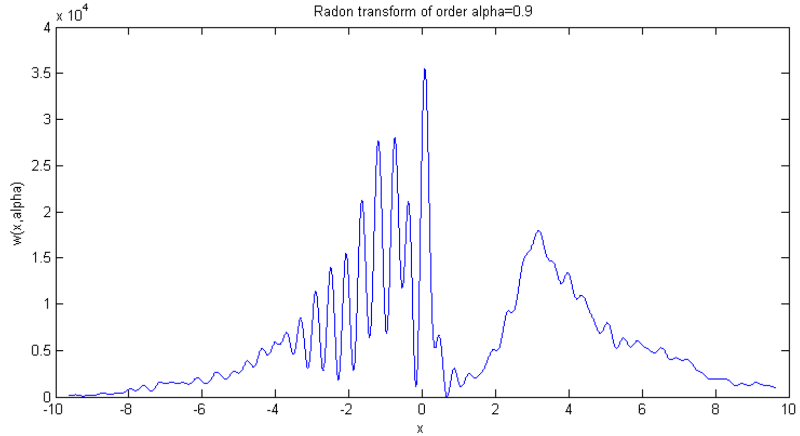

Radon transform of order alpha=0.6
w(x,alpha)
x

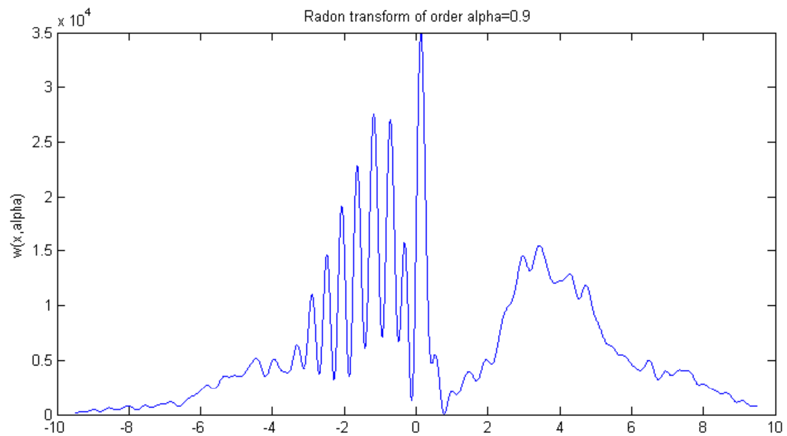

Radon transform of order alpha=0.6
w(x,alpha)
x

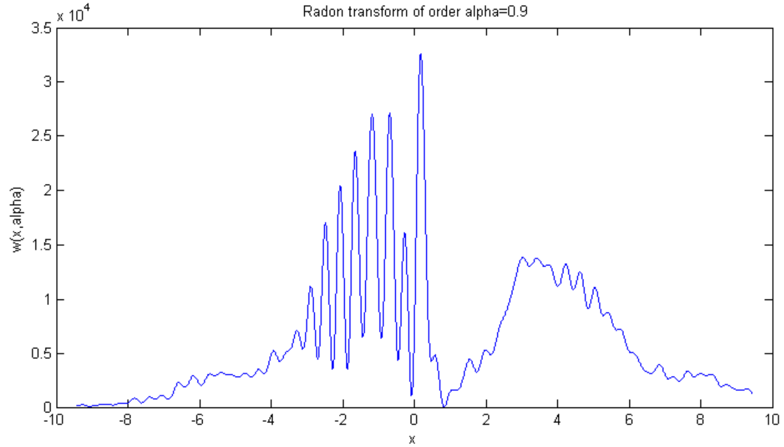

Radon transform of order alpha=0.8
x 10^4
w(x,alpha)
x

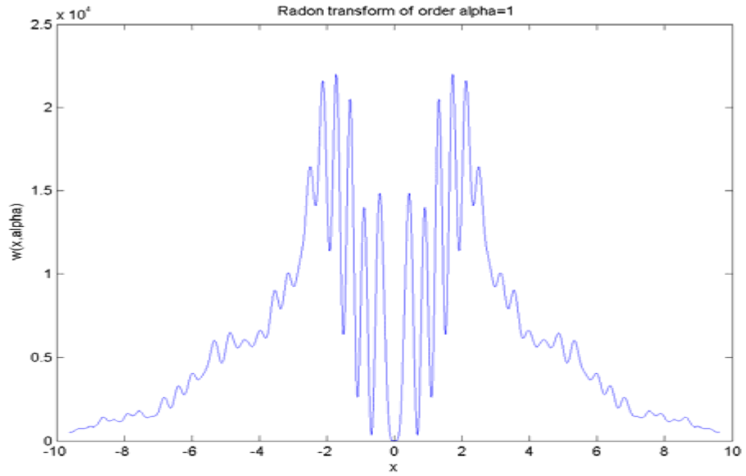

Radon transform of order alpha=0.8
x 10^4
w(x,alpha)
x

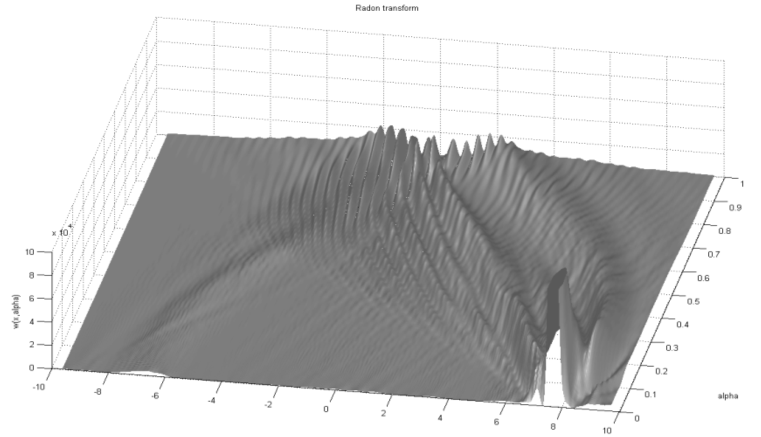

Radon transform of order alpha=0.8
x 10^4
w(x,alpha)
x

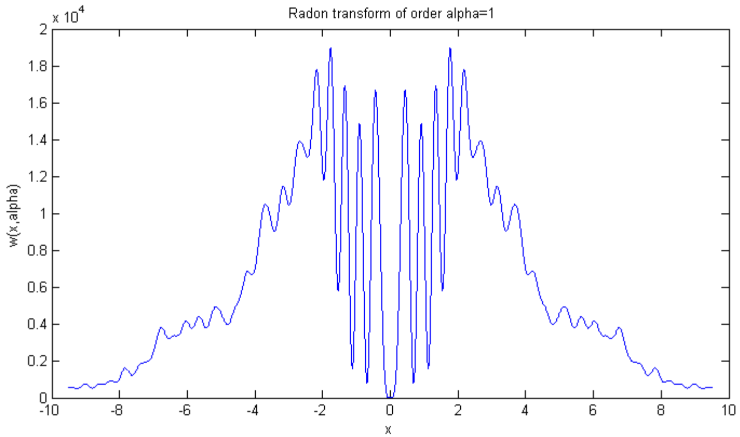

Radon transform of order alpha=0.9
x 10^4
w(x,alpha)
x

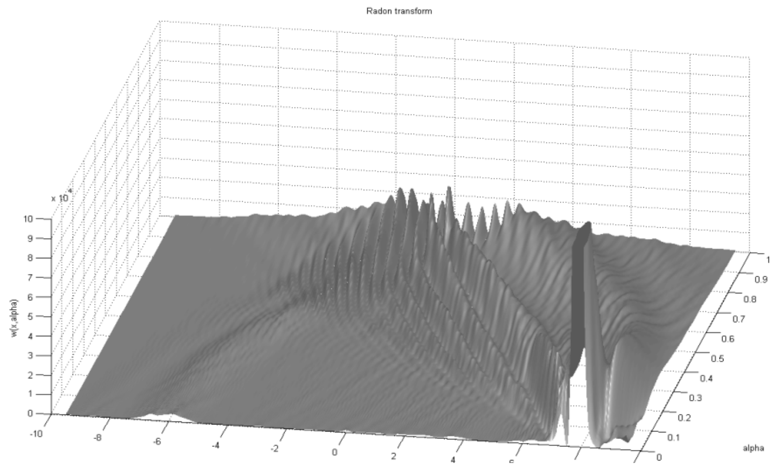

Radon transform of order alpha=0.9
x 10^4
w(x,alpha)
x

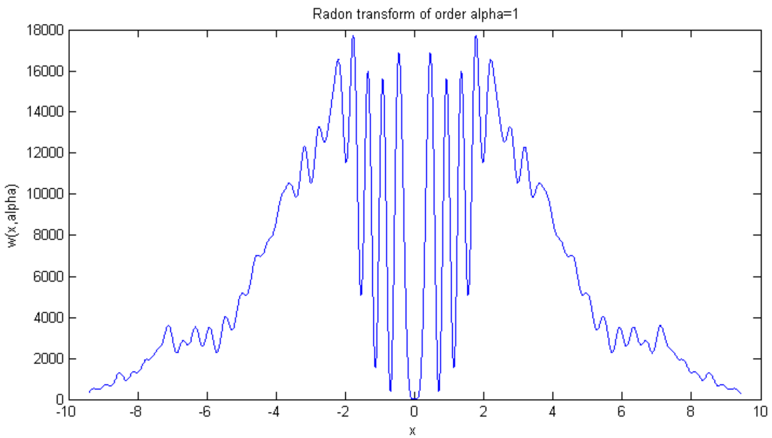

Radon transform of order alpha=0.9
x 10^4
w(x,alpha)
x

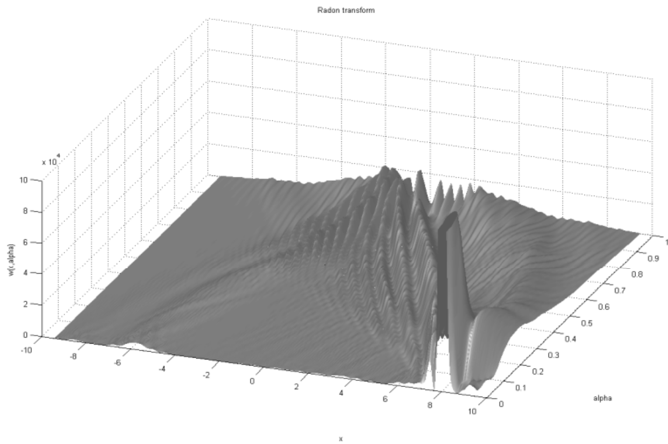

Radon transform of order alpha=1
x 10^4
w(x,alpha)
x


Radon transform
x 10^4
w(x,alpha)
alpha


Radon transform of order alpha=1
x 10^4
w(x,alpha)
x


Radon transform
x 10^4
w(x,alpha)
alpha


Radon transform of order alpha=1
w(x,alpha)
x


Radon transform
x 10^4
w(x,alpha)
alpha
x

Очищение от шумов

Signal recognition and adapted filtering by non-commutative tomography

Carlos Aguirre

R. Vilela Mendes

Rectangular signal –
Rectangular basic functions

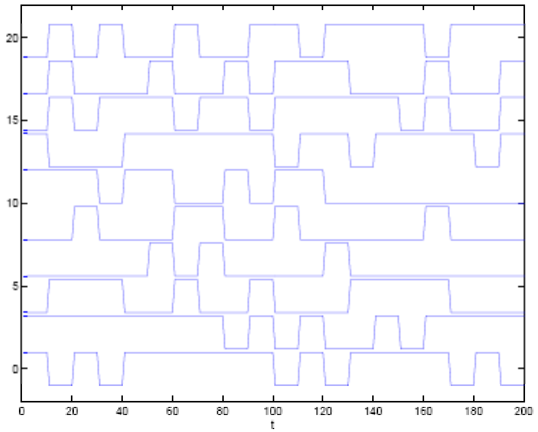


Figure 1: A set of typical signals.

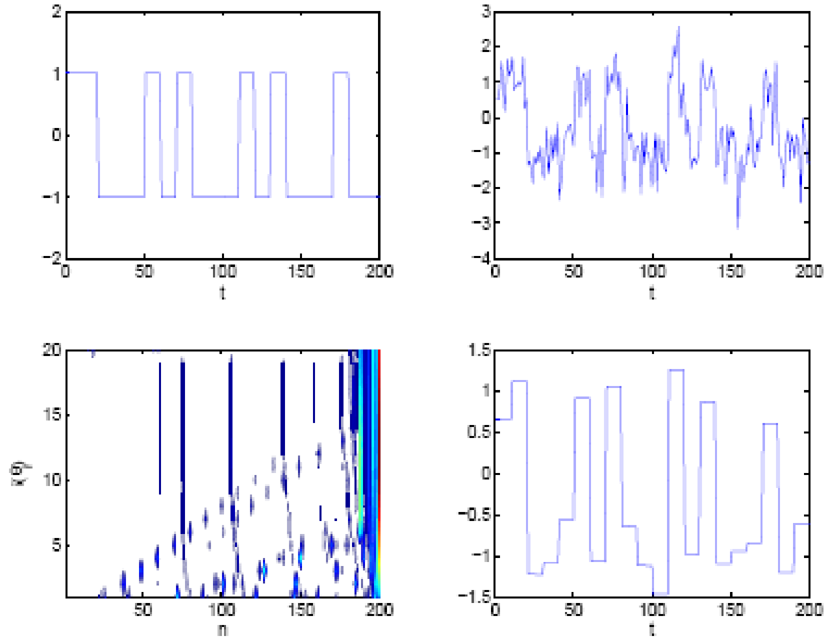


Figure 2: Signal, noisy signal, the tomogram and the projection on the eigenvectors 185 to 200 at $\theta = 19\pi/40$

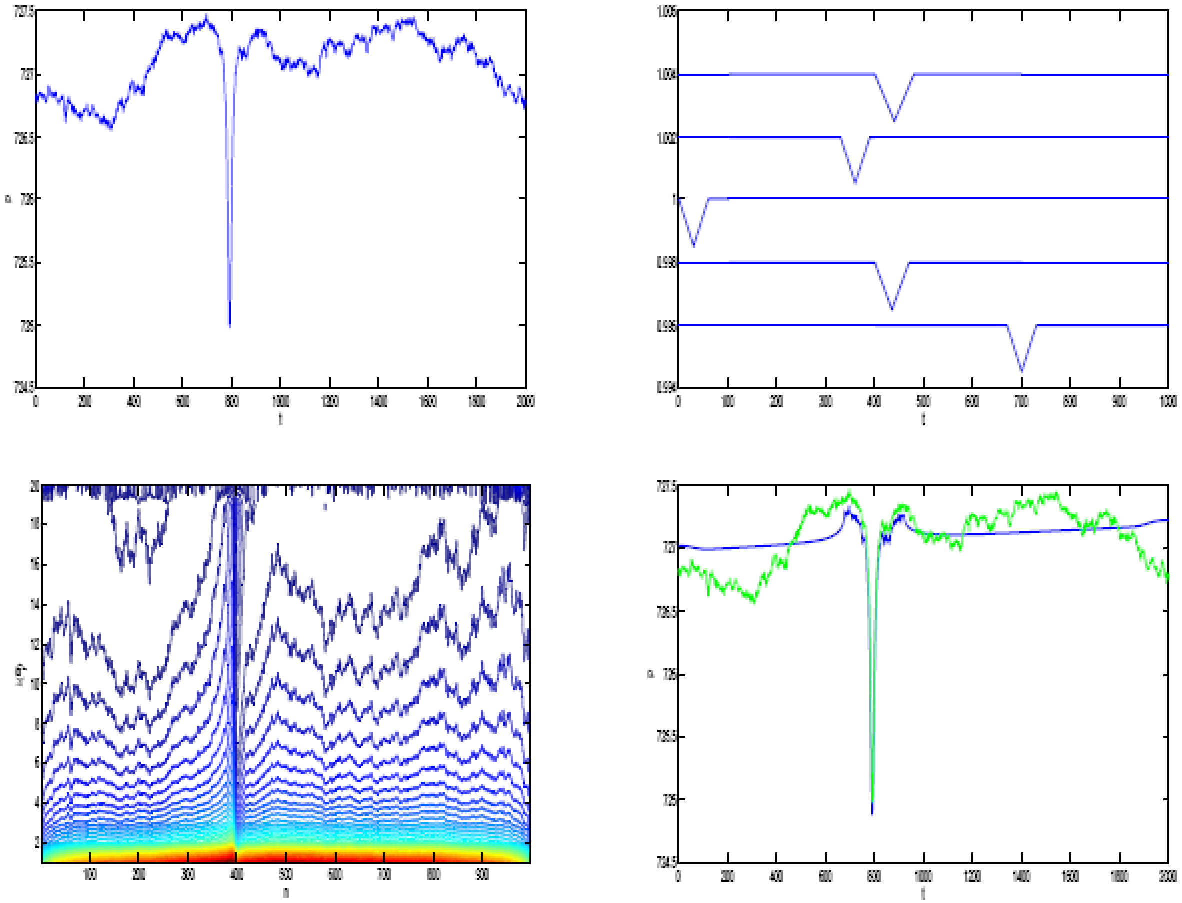


Figure 6: Signal, typical signals, the tomogram (coefs 1-999) and the projection on the eigenvectors 340 to 450 and 1000 at $\theta = 19\pi/40$

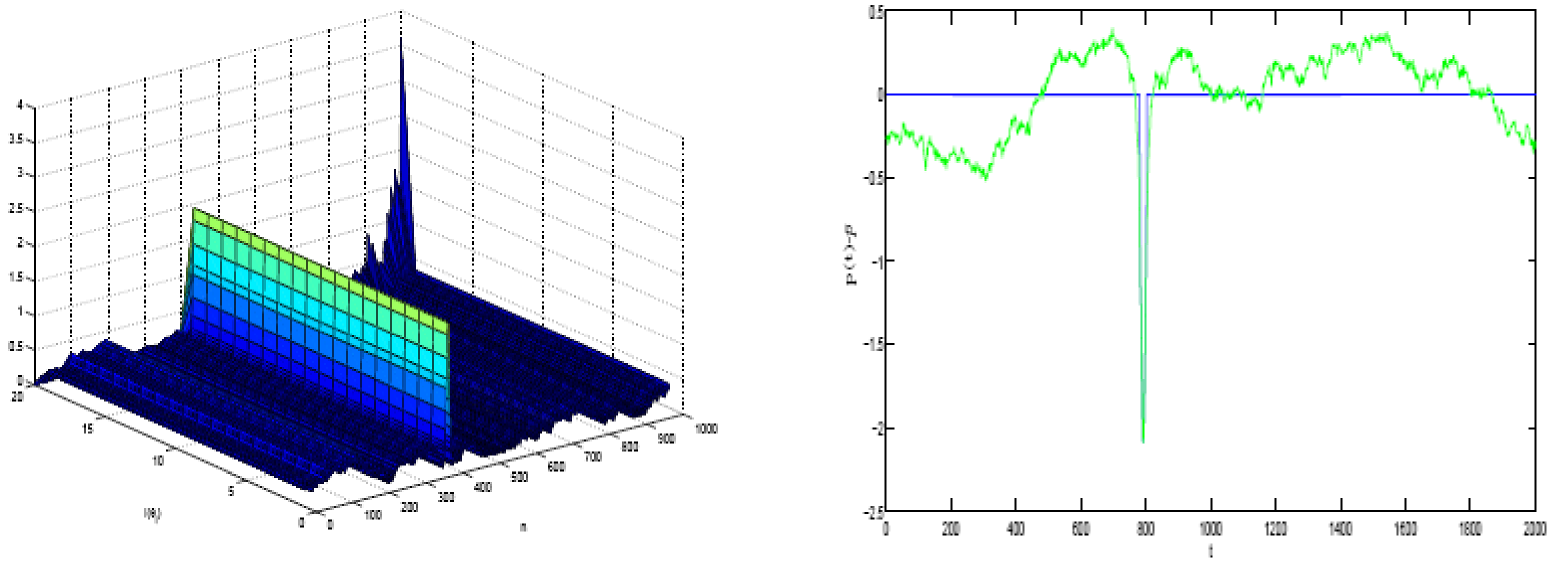
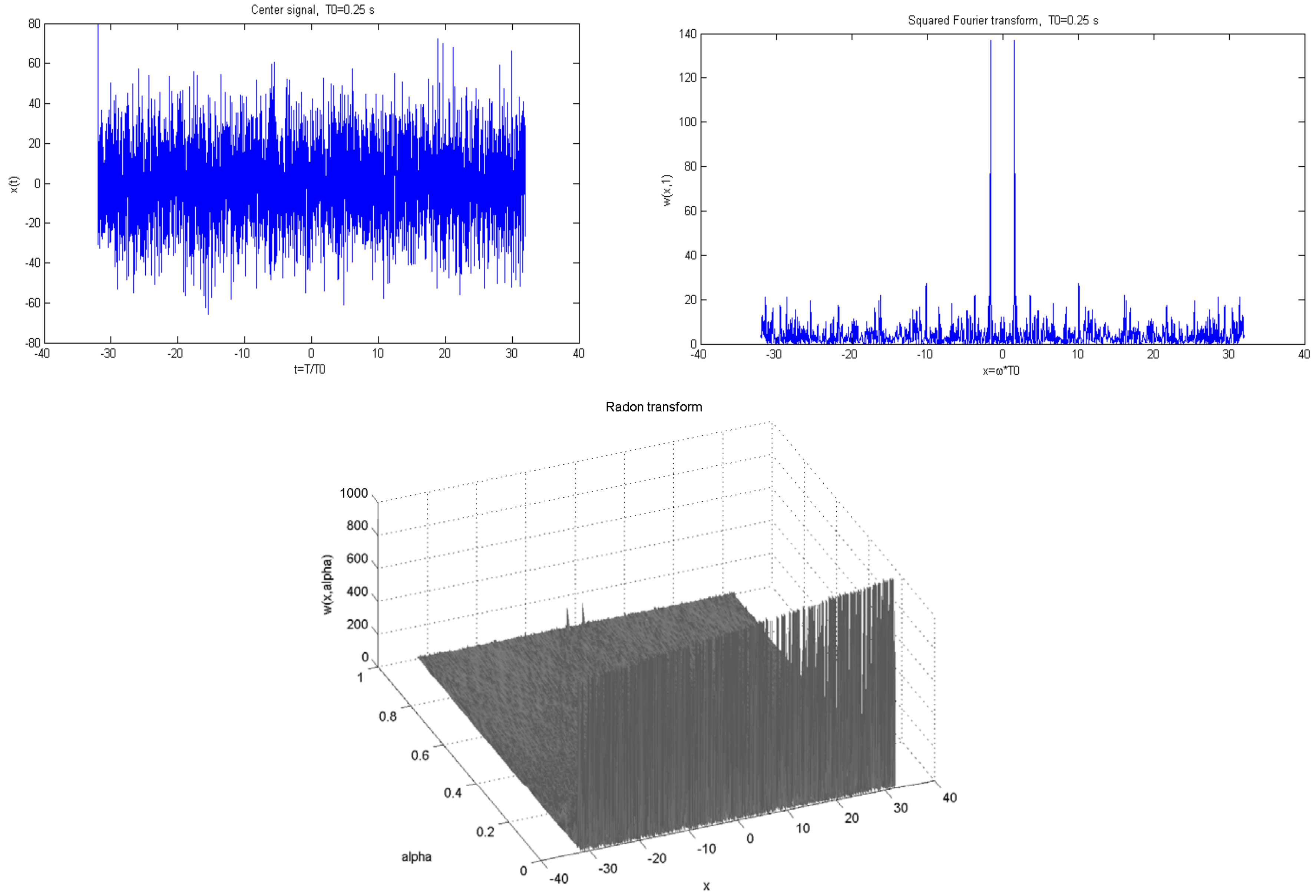
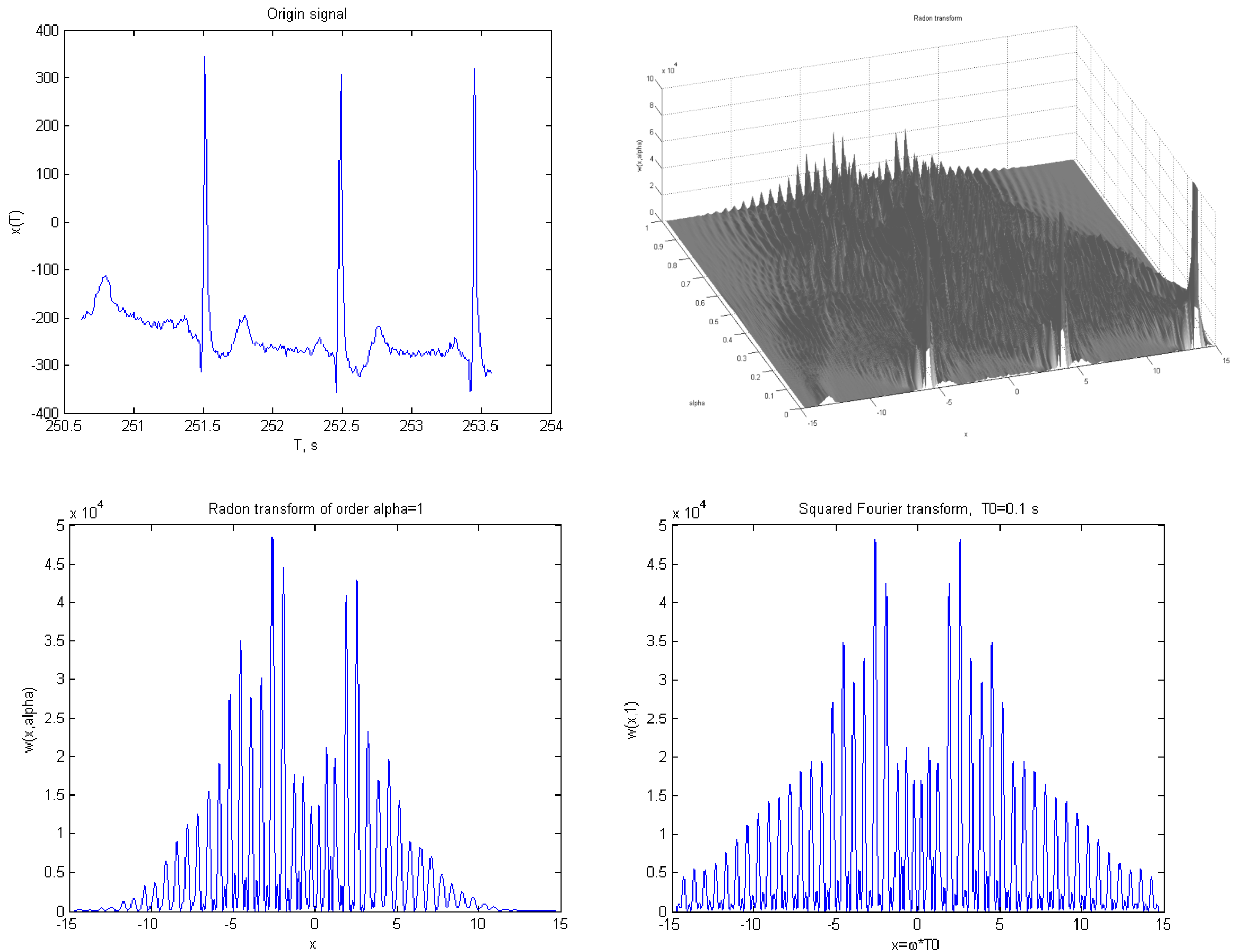


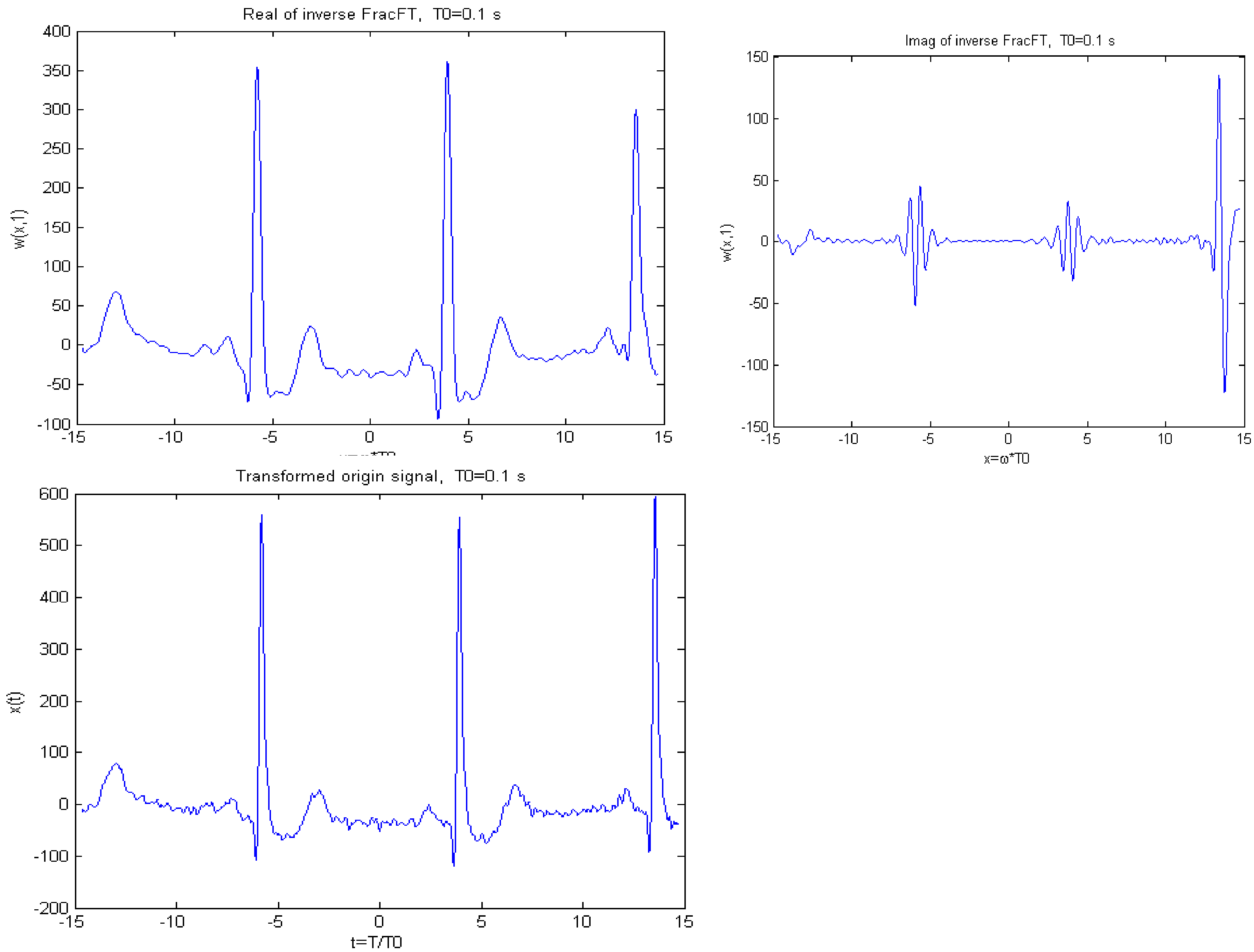
Figure 7: Tomogram for 0 mean typical signals and the projection on the eigenvectors 340 to 450 at $\theta = 19\pi/40$

Тестирование периодического сигнала с гауссовым шумом





Origin signal
x(T)
T, s
Radon transform
x 10^4
w(x,alpha)
alpha
x
Radon transform of order alpha=1
x 10^4
w(x,alpha)
x
Squared Fourier transform, T0=0.1 s
x 10^4
w(x,1)
x=ω*T0


Real of inverse FracFT, T0=0.1 s
w(x,1)
Imag of inverse FracFT, T0=0.1 s
w(x,1)
x=ω*T0
Transformed origin signal, T0=0.1 s
x(t)
t=T/T0

Спасибо

за

внимание !